Thursday, August 30, 2012 (Day 3)

Observe that the ~~sy~~ symbol $\lim_{x \to c} f(x) = L$

tells us that the graph of f appears to be heading for the location $(x, y) = (c, L)$

Another computational example

let $f(x) = \dfrac{x - 11}{|x - 11|}$

(A) find $f(11)$

Solution: $f(11) = \dfrac{11 - 11}{|11 - 11|} = \dfrac{0}{|0|} = \dfrac{0}{0}$ undefined

This tells us that graph of f has no y-value at $x = 11$.

(B) find $\lim\limits_{x \to 11^-} f(x)$

(C) find $\lim\limits_{x \to 11^+} f(x)$

(D) find $\lim\limits_{x \to 11} f(x)$

Start by exploring f(x) a bit. Come up with alternate way of writing f(x)

Plug in some x-values

x	y
0	$f(0) = \frac{0-11}{\lvert 0-11 \rvert} = \frac{-11}{\lvert -11 \rvert} = \frac{-11}{11} = -1$
9	$f(9) = \cdots\cdots = \frac{-2}{\lvert -2 \rvert} = \frac{-2}{2} = -1$
10	$f(10) = \frac{-1}{\lvert -1 \rvert} = \frac{-1}{1} = -1$
11	$f(11) = \frac{0}{0}$ = undefined
12	$f(12) = \frac{12-11}{\lvert 12-11 \rvert} = \frac{1}{\lvert 1 \rvert} = 1$
13	$f(13) = \frac{13-11}{\lvert 13-11 \rvert} = \frac{2}{\lvert 2 \rvert} = \frac{2}{2} = 1$

Conclude that

$$f(x) = -1 \quad \text{if } x < 11$$

$$f(x) = \text{undefined} \quad \text{if } x = 11$$

$$f(x) = 1 \quad \text{if } x > 11$$

So f is a "piecewise-defined function"

These are commonly abbreviated

$$f(x) = \begin{cases} -1 & \text{if } x < 11 \\ \text{undefined} & \text{if } x = 11 \\ 1 & \text{if } x > 11 \end{cases}$$

Now do the limits

(B) $\lim_{x \to 11^-} f(x) = \lim_{x \to 11^-} -1 = -1$

because $f(x) = -1$ for $x < 11$

By theorem 2.1

(C) $\lim_{x \to 11^+} f(x) = \lim_{x \to 11^+} 1 = 1$

because $f(x) = 1$ for $x > 1$

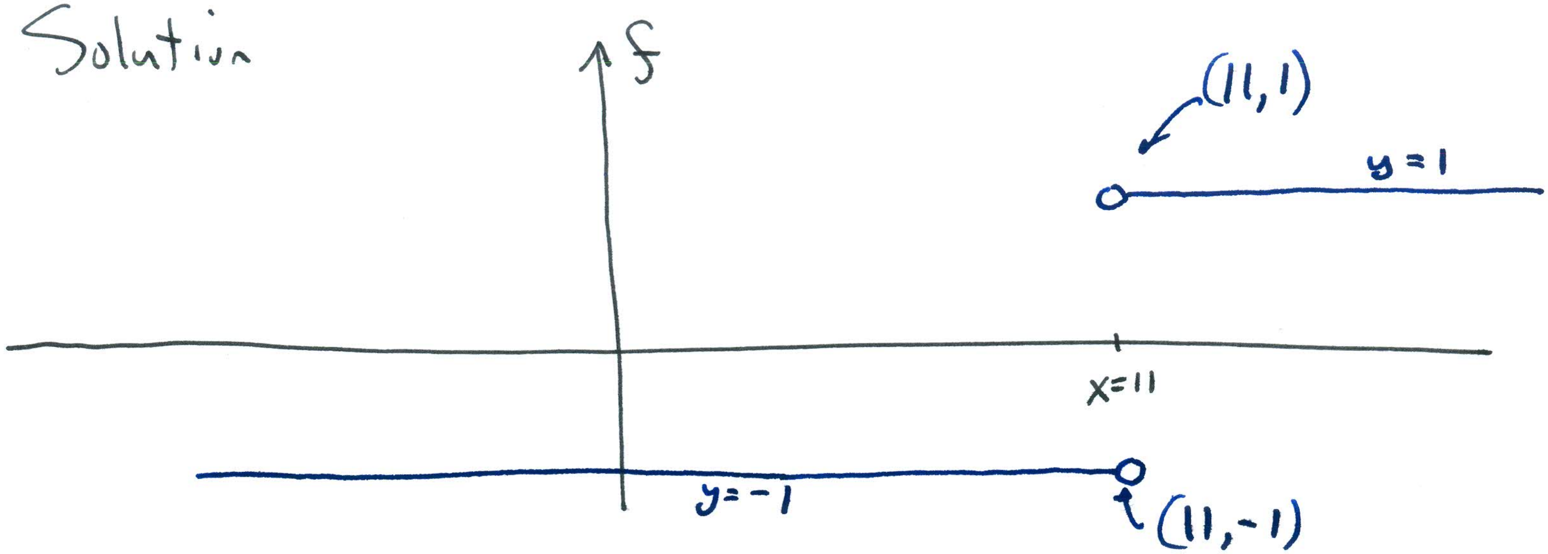
By Theorem 2.1

(D) $\lim_{x \to 11} f(x) =$ Does not exist because

the answers to (B), (C) don't match.

(E) ~~e~~explain answers to A, B, C using a graph of f.

Solution



Observe: • no y-value at $x=11$

• graph appears to be heading for $(x,y)=(11,-1)$ from the left.

That is, $\lim\limits_{x\to 11^-} f(x) = -1$

• graph appears to be heading for $(x,y)=(11,1)$ from the right.

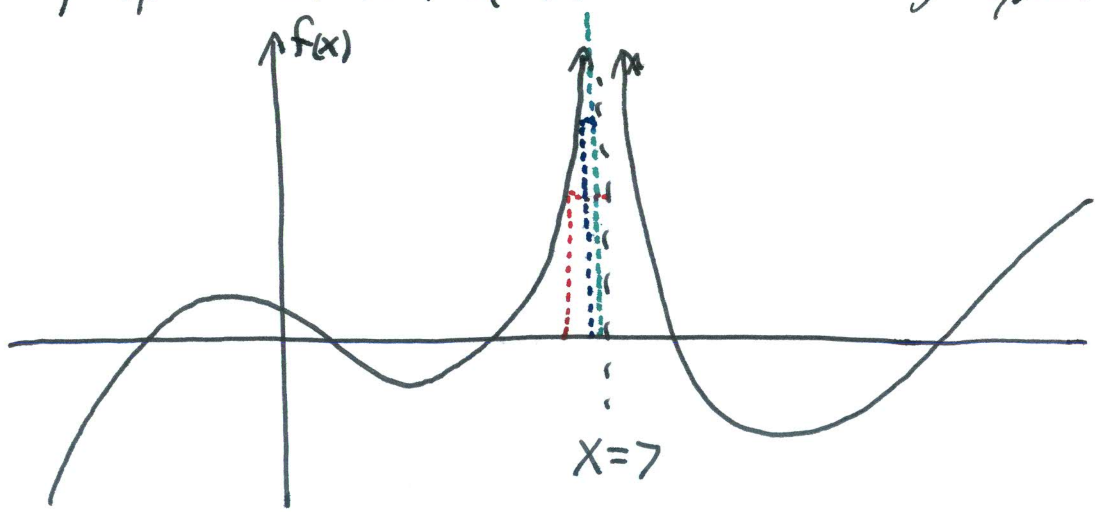
that is $\lim\limits_{x\to 11^+} f(x) = 1$

Section 3-2 Limits Involving Infinity

Today Graphical Approach (function f given by graph, not by a formula)

Vertical Asymptotes

Example of a graph with a vertical asymptote at $x = 7$

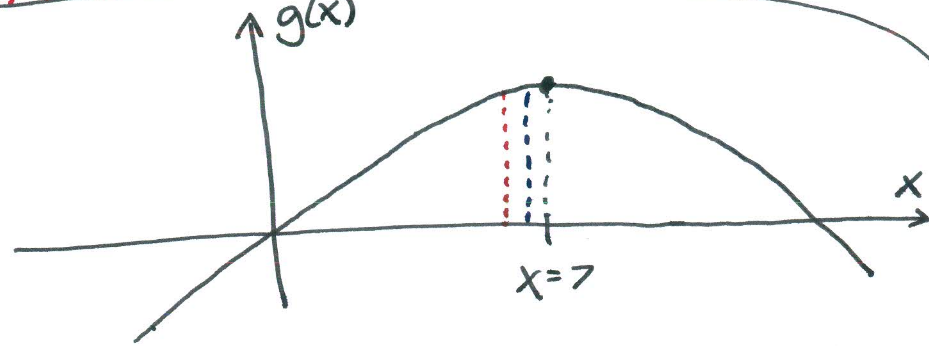


Describe in words what is happening near the asymptote.

☆ { "When x gets closer + closer to 7 but not equal to 7, the y-values get more + more positive without bound."

Q: why do we need to say "without bound"?

How about this graph?



Notice,

as x gets closer + closer to 7 but not equal to 7

the y-values are <u>also</u> getting more + more positive

but in this case, the y-values are bounded

Abbreviation for words in sentence ☆.

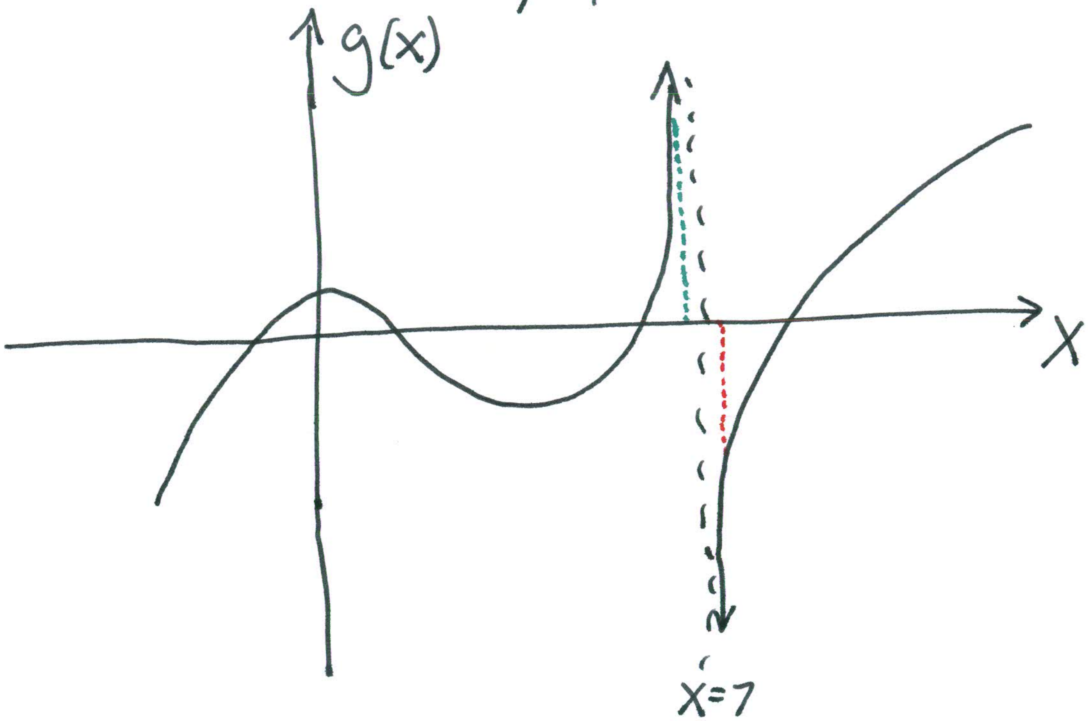
"As x approaches 7, y goes to infinity."

"As $x \longrightarrow 7$, $y \longrightarrow \infty$."

"The limit, as x approaches 7, of f(x) is infinity."

"$\lim\limits_{x \to 7} f(x) = \infty$"

Consider another graph with vertical asymptote at $x=7$



Description, in words, of what is happening near ~~at~~ $x=7$.

As x gets closer & closer to 7 from the left (but not equal to 7) the y-values get more & more positive, without bound.

abbreviation $$\lim_{x \to 7^-} g(x) = \infty$$

As x gets closer & close to 7 from the right (but not equal to 7) the y-values ~~are~~ get more & more negative without bound.

abbreviation

$$\lim_{x \to 7^+} g(x) = -\infty$$

Combine these symbols

$$\lim_{x \to 7} g(x) =$$ Does not exist because the left & right limits don't match.

(Notice bad writing in book)

graph

$x=7$

Back in Section 3-1

$\lim_{x \to 7^-} f(x)$ DNE (because in Section 3-1, limits could only be <u>numbers</u>)

$\lim_{x \to 7^+} f(x)$ DNE

$\lim_{x \to 7} f(x)$ DNE

Now in section 3-2

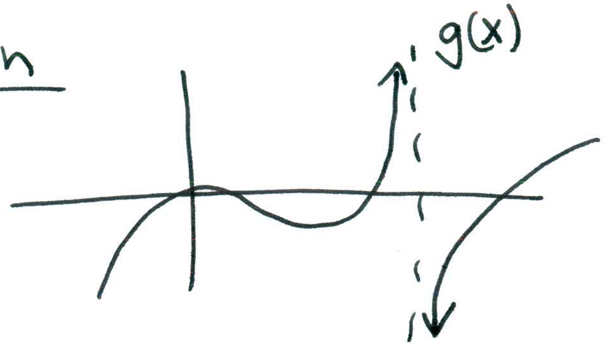
$\lim_{x \to 7^-} f(x) = \infty$

$\lim_{x \to 7^+} f(x) = \infty$

$\lim_{x \to 7} f(x) = \infty$.

The limit <u>does</u> exist! (In spite of what the book says)

Graph



Back in section 3-1

$\lim_{x \to 7^-} g(x)$ DNE

$\lim_{x \to 7^+} g(x)$ DNE

$\lim_{x \to 7} g(x)$ DNE

Now in section 3-2

$$\lim_{x \to 7^-} g(x) = \infty$$

$$\lim_{x \to 7^+} g(x) = -\infty$$

$$\lim_{x \to 7} g(x) = \text{DNE}$$