# Monday, Sept 10, 2012 (Day 6)

## Section 3-4 The Derivative

Today: Rates of Change

Series of examples involving the function

$$f(x) = \underbrace{-x^2 + 6x - 5}_{} = \underbrace{-(x-1)(x-5)}_{\text{factored form}}$$

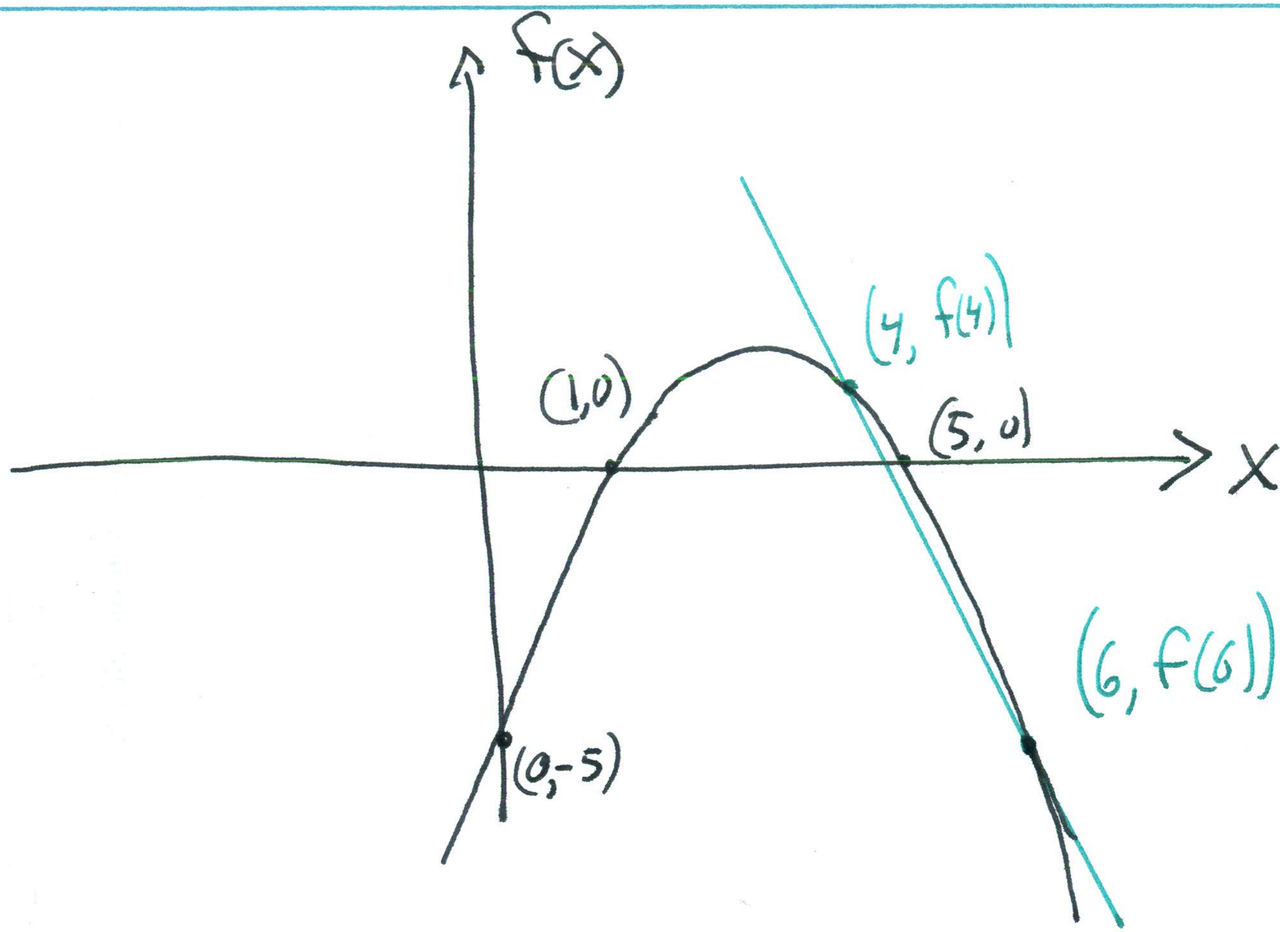
(A) Draw graph of $f$

Standard form of function tells us

- graph is a parabola (because $x^2$ term)
- upside down (negative leading coefficient $-x^2$)
- y-intercept at $(x,y) = (0,-5)$

Factored form of the equation tells us

- x-intercepts at $(x,y) = (1,0)$ and $(5,0)$




(B) Draw secant line through the point $(x,y) = (4, f(4))$ and $(x,y) = (6, f(6))$ Solution: green line

(C) Find secant line slope.

Solution $m = \frac{\Delta y}{\Delta x} = \frac{f(6) - f(4)}{6 - 4}$

~~Ex~~ We need the y-values

$$f(6) = -(6-1)(6-5) = -(5)(1) = -5$$

↑ substitute x=6 into f(x)

$$f(4) = -(4-1)(4-5) = -(3)(-1) = +3$$

↑ substitute $x=4$ into f(x)

Use these to find slope m

$$m = \frac{f(6)-f(4)}{6-4} = \frac{(-5)-(3)}{6-4} = \frac{-8}{2} = -4$$

Slope $m = -4$

## Introduce Average Rate of change

See Reference 7 in Course Packet

We see that what we just did was to compute the "Average rate of change of f from $x=4$ to $x=6$."

The result was $m=-4$

(D) find Average rate of change of f from $x = 4$ to $x = 4+h$

~~Re~~ Solution we need to find

$$m = \frac{f(4+h) - f(4)}{(4+h) - 4} = \frac{f(4+h) - f(4)}{h}$$

these cancel

we alread found $f(4) = 3$ awhile ago.

What is $f(4+h)$?

It means substitute $4+h$ into f everywhere there was ~~one~~ an x.

It is helpful to first ~~fo.~~ get the empty version of the function f.

$$f(x) = -x^2 + 6x - 5$$

$$f(\quad) = -(\quad)^2 + 6(\quad) - 5 \quad \text{empty version}$$

now put in the 4+h

$$f(4+h) = -(4+h)^2 + 6(4+h) - 5$$

$$= -(16+4h+4h+h^2) + 24 + 6h - 5$$

$$= -(16+8h+h^2) + 24 + 6h - 5$$

$$= -16 - 8h - h^2 + 24 + 6h - 5$$

$$= -h^2 - 2h + 3$$

Now assemble the expression for $m$.

these cancel

$$m = \frac{f(4+h) - f(4)}{h} = \frac{(-h^2 - 2h + 3) - (3)}{h}$$

$$= \frac{-h^2 - 2h}{h}$$

factor out an $h$ in numerator

$$= \frac{h(-h-2)}{h}$$

these cancel

$$= -h - 2.$$

(E) Find the "Instantaneous rate of Change of f at $x=4$."

Solution From Reference 7, we see that we need to compute this quantity

$$m = \lim_{h\to 0} \frac{f(4+h) - f(4)}{h}$$

We found the fraction ~~numerator~~ in previous question.

we found $\frac{f(4+h)-f(4)}{h} = -h-2$.

So now, we just take the limit to find m.

$$m = \lim_{h\to 0} -h-2 \underset{\substack{\text{substitute} \\ \text{in } h=0}}{=} -0-2 = -2$$

So instantaneous rate of change is $m=-2$.

graphical significance:

The number $m=-2$ is the slope of the line tangent to graph of $f$ at the point where $x=4$.