Monday, September 24, 2012 (Day 12)

Continuing Section 3-7 Marginal Analysis

Today, we will use "marginal quantities" to estimate changes in quantities.

Underlying theory

Suppose you have a function f

and two different known x values x_1 and x_2.

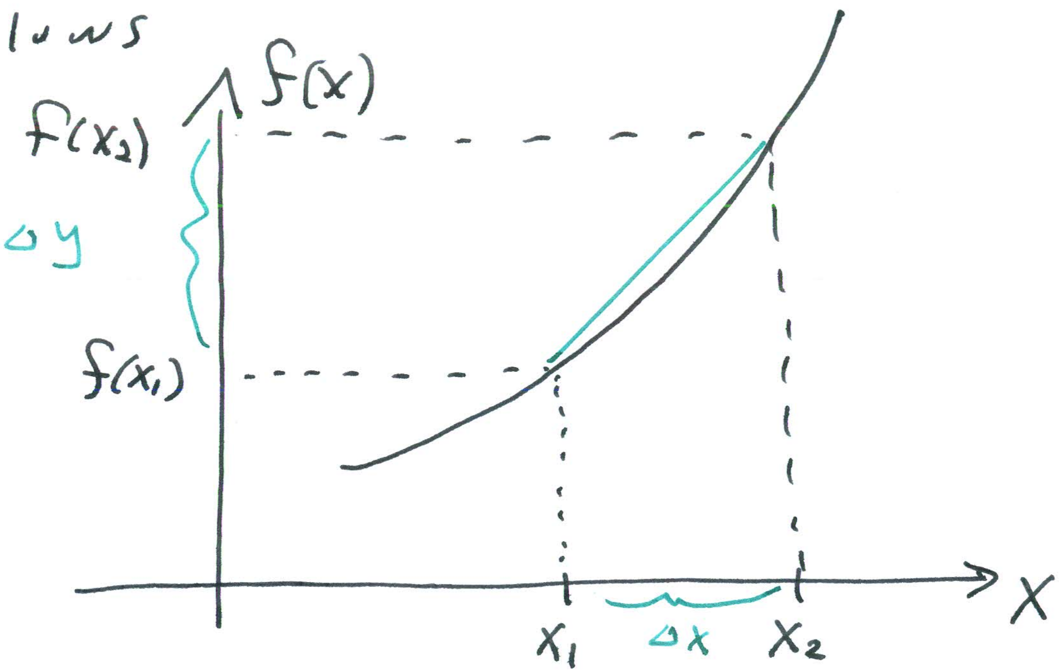
Change in input is $\Delta x = x_2 - x_1$

Question: what is the resulting change in output?

That is, what is Δy?

Answer: $\Delta y = y_2 - y_1 = f(x_2) - f(x_1)$

On a graph, these quantities would look as follows

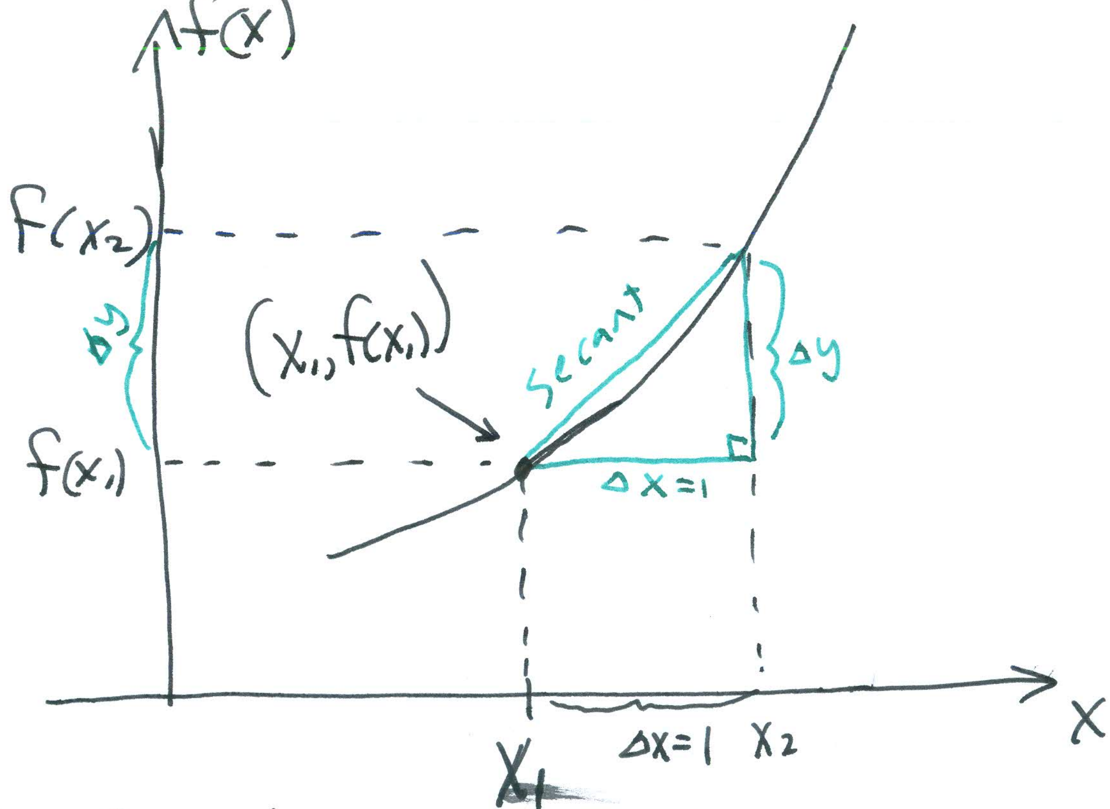


Observe the secant line has slope

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Suppose that we have a special case where x_1 is known, and x_2 is $x_2 = x_1 + 1$

In other words, x_1 is known, and $\Delta x = 1$

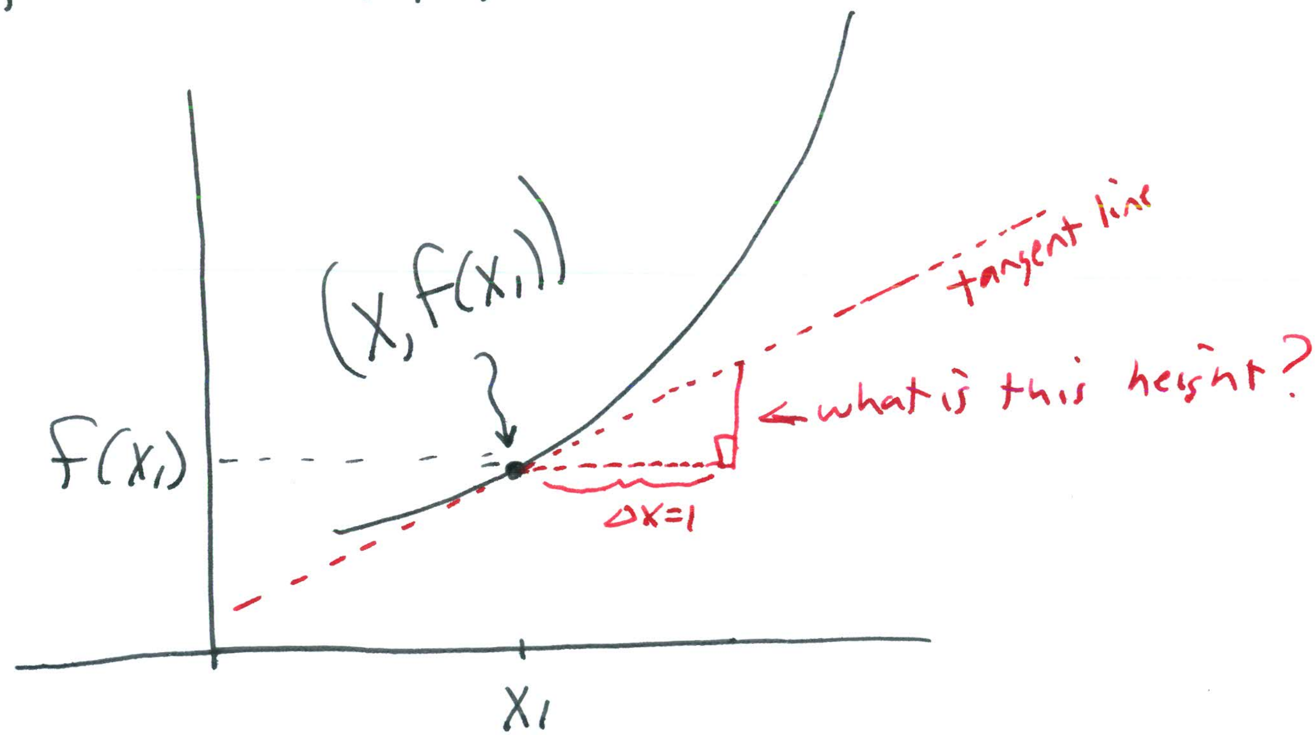


In this case, secant line has slope

$$m = \frac{\Delta y}{\Delta x} = \frac{\Delta y}{1} = \Delta y = f(x_2) - f(x_1)$$

$$= \text{change in } y$$

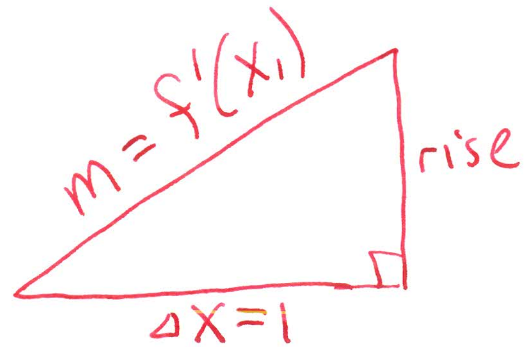
Consider the line that is <u>tangent</u> to graph of f at x_1.



remember the <u>tangent</u> <u>line</u> has slope

$$m = f'(x_1)$$

Imagine a right triangle using this tangent line as hypotenuse and having a base $\Delta x = 1$



Slope $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{\text{rise}}{1}$

$m = \text{rise}$

That is $\text{rise} = m = f'(x_1)$

for the red triangle.

From these drawings

rise in red triangle $\approx$ Δy in ~~red~~ green triangle

approx but not equal

$f'(x_1) \approx \text{change in } y = f(x_2) - f(x_1)$

We will use this fact to approximate changes in y that result from $\Delta x = 1$

Example #1 Exercise 3-7 # 26

A company manufactures guitars. The cost of producing a batch of x guitars is $C(x) = 1000 + 100x - .25x^2$ dollars.

(A) Book's question: "Find the cost of producing the 51^{st} guitar"

My Rewording of the question

If the batch size changes from $x = 50$ to $x = 51$ guitars, what will be the change in cost of producing a batch?

Solution

$$\Delta \text{Cost} = C(51) - C(50)$$
$$= \left(1000 + 100(51) - .25(51)^2\right) - \left(1000 + 100(50) - .25(50)^2\right)$$
$$= 5449.75 \qquad - \quad 5375$$
$$= \$74.75$$

Book wording.

(B) "Use marginal analysis to <u>approximate</u> the cost of the 51^{st} guitar"

<u>My rewording:</u>

Use marginal analysis to find the <u>approximate change</u> in cost of a batch of guitars if the batch size changes from 50 to 51.

<u>Solution</u> We will use fact that

$$\text{change} \approx C'(x_1)$$

So, find $C'(x)$, then substitute in $x_1 = 50$.

$$C(x) = 1000 + 100x - .25x^2$$

$$C'(x) = \frac{d}{dx}\left(1000(1) + 100x - .25x^2\right)$$

$$= 1000\left(\frac{d}{dx}1\right) + 100\left(\frac{d}{dx}x\right) - .25\left(\frac{d}{dx}x^2\right)$$

$$C'(x) = 1000(0) + 100(1) - .25(2x)$$
$$= 100 - .5x$$

So $C'(x_1) = C'(50) = 100 - .5(50)$
$$= 100 - 25$$
$$= 75$$

This is an approximation of the change in cost of a batch of guitars when batch size changes from $x=50$ to $x=51$.

<u>Example #2</u> (3-7 #36)

A company manufactures lamps. The price p (in dollars) and demand x are related by the equation

$$X = 1000 - 20p$$

The price demand equation

Solve this equation for p in terms of x and find the domain

Solution

$$X - 1000 = -20p$$

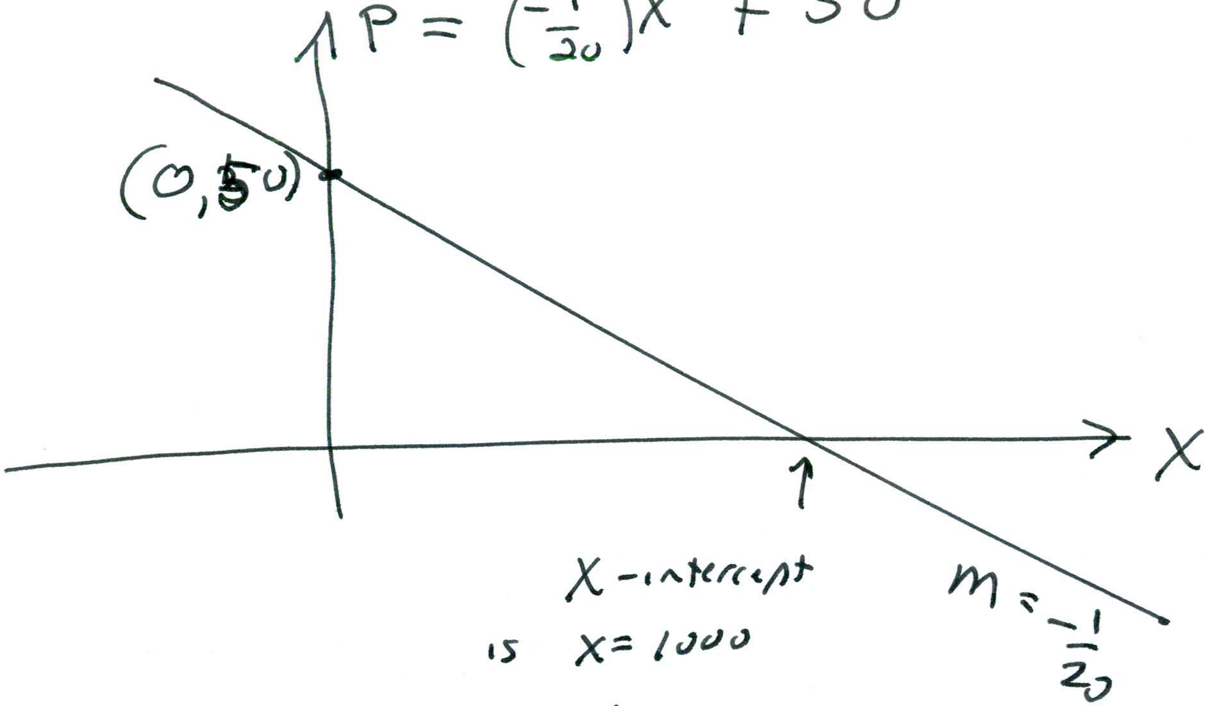
$$\frac{X - 1000}{-20} = p$$

$$p = -\frac{x}{20} + \frac{1000}{20}$$

$$p = -\frac{x}{20} + 50$$

what is the domain?? That is, what are the allowed values of x?

Try graphing the equation to find out



because
Set $p=0$ and solve for x

$$0 = \left(-\frac{1}{20}\right)x + 50$$

$$\left(+\frac{1}{20}\right)x = 50$$

$$x = 20(50) = 1000$$

So domain is $0 \le x \le 1000$