Tuesday, September 25, 2012 (Day 13)

Review Yesterday's Discussion of Marginal Analysis.

Given a function f

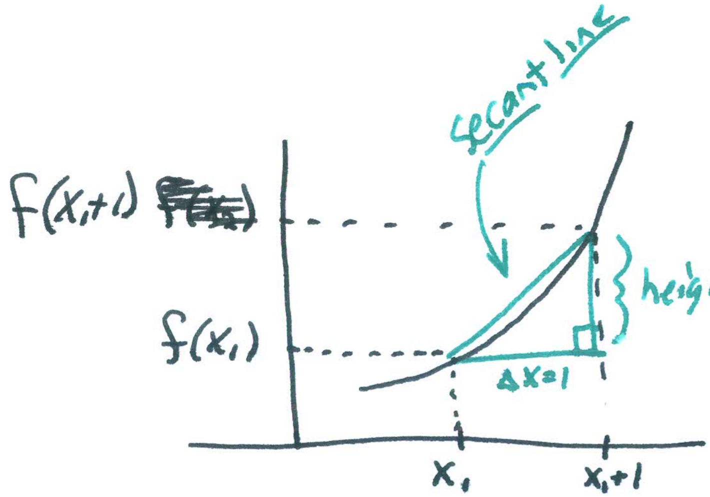
Given a known x-value x_1

Question: What is the *exact* change in y-value, when the x-value changes by $\Delta x = 1$?

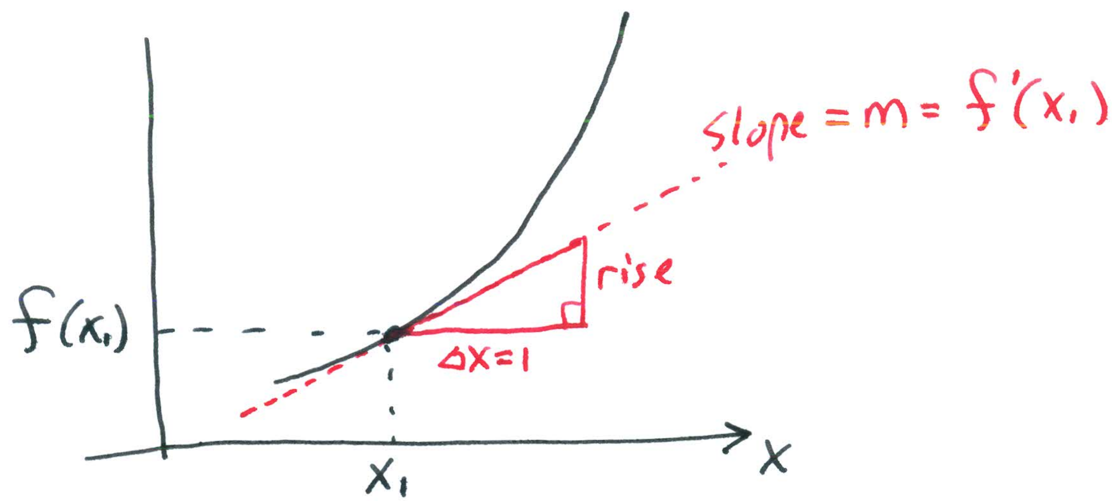
Answer: Exact change $= \Delta y$

$$= f(x_2) - f(x_1)$$

$$= f(x_1 + 1) - f(x_1)$$



We can also draw a triangle using the line that is <u>tangent</u> at the point $(x,y) = (x_1, f(x_1))$



$$m = f'(x_1) = \frac{\text{rise}}{\text{run}} = \frac{\text{rise}}{1} = \text{rise}$$

$$\text{So} \quad \text{rise} = f'(x_1)$$

Then we observed

exact change Δy on green triangle $\approx$ rise on red triangle

$$\boxed{\Delta y \approx f'(x_1)}$$

We explored this fact in an example about guitars.

Resuming 2nd Example, about lamps

(B) (Continuing example)

Find the Revenue function ~~for~~ $R(x)$ and its domain

Solution:

Revenue $\quad R(x) = x \cdot p$ from yesterday

$$= x \cdot \left(\left(-\tfrac{1}{20}\right) x + 50 \right)$$

$$= \left(-\tfrac{1}{20}\right) x^2 + 50x$$

Domain is $0 \leq x \leq 1000$ because the equation for p from yesterday was only valid for $0 \leq x \leq 1000$.

(c) Find the "Marginal Revenue at a Production Level of 400 units"

and interpret the results

Solution this means find $R'(400)$

Strategy: find $R'(x)$

Substitute in $x=400$

$$R(x) = \left(-\frac{1}{20}\right)x^2 + 50x$$

$$R'(x) = \frac{d}{dx}\left(\left(\frac{-1}{20}\right)x^2 + 50x\right)$$

$$= \left(-\frac{1}{20}\right)\left(\frac{d}{dx}x^2\right) + 50\left(\frac{d}{dx}x\right)$$

$$= \left(-\frac{1}{20}\right)(2x) + 50(1)$$

$$R'(x) = -\frac{x}{10} + 50$$

Now substitute in $x=400$

$$R'(400) = -\frac{400}{10} + 50 = \cancel{10}\ -40+50 = 10.$$

~~Graph~~ Interpretation

Graphically, this tells us that the line tangent to graph of $R(x)$ at $x=400$ has slope $m=10$.

In terms of Marginal analysis, this tells us that if the size of a batch of lamps ~~gets~~ changes from $x=400$ to $x=401$, the resulting change in revenue would be approximately \$10.

(D) Find the marginal revenue at a production level of 650 and interpret the results.

Solution $\cancel{R'(60)}$ $R'(650) = -\frac{650}{10} + 50 = -65+50 = -15$

This tells us that if batch size changes from $x=650$ to $x=651$ lamps, the revenue will decrease by roughly \$15.

Chapter 4

Section 4-1 The constant "e" and compound interest.

We will discuss three kinds of bank account interest.

- Simple Interest
- Periodically Compounded Interest
- Continuously Compounded Interest.

Simple Interest

Interest is only earned on the initial deposit.

Define

P = initial deposit (the Principal)

r = interest rate (expressed as a decimal)

t = time (in years) that money was in account

A = account balance.

Simple Interest Formula

$$A = \text{Principal} + \text{Interest}$$

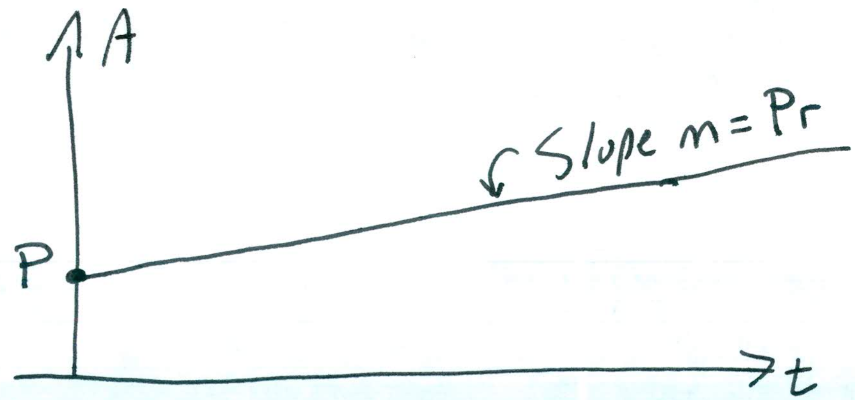
$$= P + Prt$$

factor this to make it look simpler

$$A = P(1 + rt) \quad \text{Simple Interest Formula}$$

Make a graph of A-vs-t for simple interest

$$A = \underbrace{(Pr)}_{\text{slope}}t + \underbrace{P}_{\text{y-intercept}}$$



Example 1

Deposit \$1000 into account with 3% simple interest. What will be balance after 5 years?

Solution

$P = 1000$

$r = .03$

$t = 5$

$A =$ unknown. ← find this

$$A = 1000(1 + (.03)5)$$

$$= 1000(1 + .15)$$

$$= 1000(1.15)$$

$$A = \$1150$$

Periodically Compounded Interest

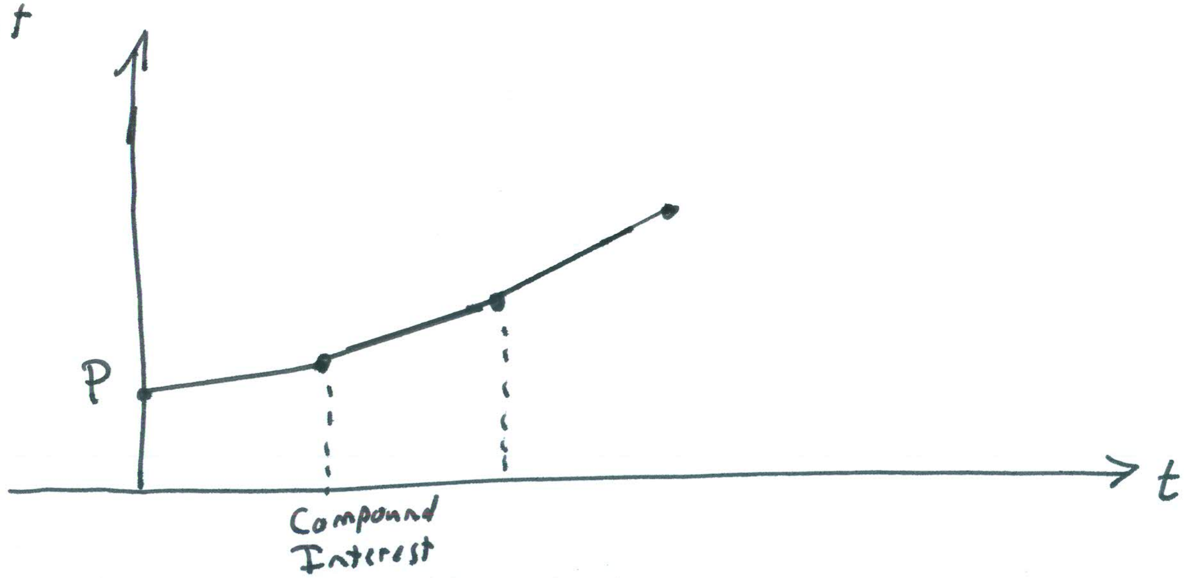
Deposit amount P

Earn simple interest for awhile, on ~~[illegible]~~ initial deposit P.

After awhile, dump the interest in with the principal

Earn simple interest for awhile, but earning interest on larger pot of money.

repeat



Periodically Compounded Interest Formula

P = Principal (initial deposit)

r = interest rate

t = time in years

m = number of times per year that the interest gets compounded.

A = account Balance.

$$\boxed{A = P\left(1 + \frac{r}{m}\right)^{mt}}$$

Periodically Compounded Interest Formula

<u>Example #2</u>

Deposit \$1000 into account with 3% interest compounded yearly. What will be balance after 5 years?

Solution $P = 1000$, $r = .03$, $t = 5$, $m = 1$ compounded yearly

$$A = 1000\left(1 + \frac{.03}{1}\right)^{1\cdot 5} = 1000(1 + .03)^5$$

$$= 1000(1.03)^5 \leftarrow \text{exact answer}$$

$\approx \$1159.\underline{27}$ approximate answer

Example #3 Same question, but compounded monthly.

Solution. $P = 1000$, $r = .03$, $t = 5$, $m = 12 \leftarrow$

$$A = 1000\left(1 + \frac{.03}{12}\right)^{12\cdot 5} \quad \text{exact answer}$$

$\approx 1161^{\underline{62}}$ approximate answer

Example #4 Same Question, compound Daily ($m = 365$)

$$A = 1000\left(1 + \frac{.03}{365}\right)^{(365\cdot 5)} \quad \text{exact}$$

$\approx \$1161^{\underline{83}}$ approximate

Notice the trend in examples 2, 3, 4

All had $P = 1000$, $r = .03$, $t = 5$

Example #2 $\quad m = 1 \implies A \approx 1159^{\underline{27}}$

Example #3 $\quad m = 12 \implies A \approx 1161^{\underline{62}}$

Example #4 $\quad m = 365 \implies A \approx 1161^{\underline{83}}$

Obvious question:

As $m \to \infty$, what is the limit of A?

That is what is

$$\lim_{m \to \infty} P\left(1 + \frac{r}{m}\right)^{m \cdot t} \quad ??$$

<u>Answer</u>

$$\lim_{m \to \infty} P\left(1 + \frac{r}{m}\right)^{m \cdot t} = Pe^{(rt)}$$

Inspired by this, we <u>invent</u> a new kind of bank account.

"Continuously-Compound Interest"

Described by the equation

$$A = Pe^{(rt)}$$

<u>Example #5</u> Deposit \$1000 into account with 3% interest compounded continuously.

What is the ~~bal~~ balance after 5 years?

Solution Use $A = Pe^{(rt)}$

$$= 1000 \cdot e^{(.03 \cdot 5)} \quad \text{exact}$$

$$\approx 1161^{\underline{83}}$$