Day 14 is Thursday, September 27, 2012

Continuing Section 4-1

Three Different Kinds of Bank Interest

Simple Interest: $A = P(1 + rt)$

Periodically Compounded Interest: $A = P\left(1 + \frac{r}{m}\right)^{m \cdot t}$

Continuously Compounded Interest: $A = Pe^{(rt)}$

Today Problems involving the formula $A = Pe^{(rt)}$

Observe the equation. $A = Pe^{(rt)}$

expresses a relationship between A, P, r, t

The equation is solved for A

We can solve this equation for the other letters.

Solve for P

$$A = Pe^{(rt)}$$

Divide both sides by $e^{(rt)}$

$$\frac{A}{e^{(rt)}} = P$$

Solve original equation for r

$$A = Pe^{(rt)}$$

Divide by P

$$\frac{A}{P} = e^{(rt)}$$

take ln( ) of both sides

$$\ln\left(\frac{A}{P}\right) = \ln\left(e^{(rt)}\right)$$

$$\ln\left(\frac{A}{P}\right) = rt$$

Divide by t

$$\frac{\ln\left(\frac{A}{P}\right)}{t} = r$$

Solve Original equation for t

$$A = Pe^{(rt)}$$

$\vdots$

Same steps as in previous example

$$\ln\left(\frac{A}{P}\right) = rt$$

Divide by r

$$\frac{\ln\left(\frac{A}{P}\right)}{r} = t$$

Summary

$A = Pe^{(rt)}$ solved for A

$P = \dfrac{A}{e^{(rt)}}$ Solved for P

$r = \dfrac{\ln(A/P)}{t}$ Solved for r

$t = \dfrac{\ln(A/P)}{r}$ Solved for t

<u>Remark</u>: Doing Math "in place" is a bad idea.

- unreliable
- impossible to check
- impossible to read

<u>Example</u> Solve this equation for r

$$\frac{\ln\left(\frac{A}{P}\right)}{t} = \frac{\cancel{P}\,\cancel{e}^{(r\cancel{t})}}{\cancel{P}}$$

Examples involving selecting the best equation to use.

Example #1 Deposit \$885 into account with 4.3% interest compounded continuously. How long until the balance is \$1500 (give exact answer, and a decimal approximation)

Solution

$P = 885$

$r = .043$

$A = 1500$

$t =$ unknown Find t.

Use the equation that is solved for t.

$$t = \frac{\ln(A/P)}{r} = \frac{\ln(1500/885)}{.043} \quad \text{exact answer}$$

$$\approx 12.3 \text{ years}$$

Remark be sure to use the right equation!

Common exam problem

(A) Solve $A = Pe^{(rt)}$ for t

(B) Example #1 just done

Common solution to (B)

$$1500 = 885\, e^{(.043t)}$$

$$\frac{1500}{885} = e^{.043t}$$

$$\ln(1500/885) = .043t$$

$$\frac{\ln(1500/885)}{.043} = t$$

this student substituted numbers into the original equation

TOO HARD

NOT A SMART SOLUTION

Example#2 Deposit some money into an account that earns 5% interest compounded continuously. How long until the balance Triples?

Solution $r = .05$

$P =$ unknown

$A = 3P$ (we want to know when balance will be 3×Principal)

$t =$ unknown. Find t.

Solution

Use the equation that is solved for t.

$$t = \frac{\ln\left(\frac{A}{P}\right)}{.05} = \frac{\ln\left(\frac{3P}{P}\right)}{.05} = \frac{\ln(3)}{.05}$$ exact answer

≈ 21.97 years

≈ 22 years

Section 4-2 Derivatives of Exponential Functions

Three New Derivative Rules

1. $\frac{d}{dx} e^{(x)} = e^{(x)}$

2. $\frac{d}{dx} b^{(x)} = b^{(x)} \cdot \ln(b)$

3. $\frac{d}{dx} e^{(cx)} = c \cdot e^{(cx)}$

Examples using these Rules

(A) $f(x) = 11e^{(x)}$ find $f'(x)$

notice: $f'(x)$ is same as $f(x)$!!!

Solution

$$f'(x) = \frac{d}{dx} 11 e^{(x)} = 11 \frac{d}{dx} \underbrace{e^{(x)}}_{\text{exponential}} = 11 \cdot e^{(x)}$$

constant multiple rule

new rule (1)

(B) $f(x) = 11x^e$ find $f'(x)$

Solution

$$f'(x) = \frac{d}{dx} 11 x^e = 11 \frac{d}{dx} \underbrace{x^e}_{\text{power function}} = 11(e x^{e-1}) = 11 e \cdot x^{e-1}$$

constant multiple rule

Power Rule

(C) $f(x) = 11 \cdot e^{(12)}$ find $f'(x)$

Solution $f(x) = 11 \cdot e^{(12)}$ is constant, so $f'(x) = 0$!!

(D) $f(x) = \cancel{\ } 11 \cdot 12^{(x)}$ find $f'(x)$

<u>Solution</u>

$$f'(x) = \frac{d}{dx} 11 \cdot 12^{(x)}$$

exponential function with base b=12

<u>Solution</u> $f'(x) = \frac{d}{dx} 11 \cdot 12^{(x)} = 11 \frac{d}{dx} 12^{(x)} = 11\left(12^{(x)} \ln(12)\right)$

constant multiple rule

new rule ②

(E) $f(x) = 11 e^{(12x)}$ find $f'(x)$

<u>Solution</u>

$$f'(x) = \frac{d}{dx} 11 e^{(12x)} = 11 \frac{d}{dx} e^{(12x)} = 11 \cdot \left(12 e^{(12x)}\right)$$

constant multiple rule

new rule ③

$$= 132 e^{(12x)}$$

Tangent line Problem

Let $f(x) = 11e^{(x)} + 23x$

Find equation of the line tangent to graph of f at $x=0$.

Solution Remember that when we need the tangent line equation, we have to build this:

$$(y - f(a)) = f'(a)(x-a)$$

generic tangent line equation (from Tuesday September 18^{th} notes)

Start by getting parts

$a = 0$ (this is the x-coord of point of tangency)

$f(a) = f(0) = 11 \cdot e^{(0)} + 23 \cdot (0) = 11 \cdot 1 + 0 = 11$

(this is the y-coord. of point of tangency)

$$f'(x) = \frac{d}{dx}\, 11 e^{x} + 23 x$$

$$= 11 \frac{d}{dx} e^{x} + 23 \frac{d}{dx} x$$

$$= 11 (e^{x}) + 23(1)$$

$$= 11 e^{x} + 23$$

$f'(a) = f'(0) = 11 \cdot e^{(0)} + 23 = 11 \cdot 1 + 23 = 11 + 23$

$= 34$ this is the slope of tangent line

Now substitute parts into equation

$$(y-11) = 34(x-0)$$

$$y-11 = 34x$$

$$y = 34x + 11$$

equation for tangent line.

<u>Remark</u>: Notice that the tangent line equation <u>is</u> a <u>line</u> equation.

That is, it has the form

$$y = mx + b.$$