Day 16 is Tuesday, October 2, 2012

Please sit in groups of three.

Working on New Class Drill

Derivatives of Functions
Containing Logarithms.

Derivatives of Functions Containing Logarithms

2012 - 2013 Fall Semester MATH 1350 (Barsamian) Class Drill
(based on Section 4-2 Example 2 and Example 3 and suggested exercise 4-2#19)

(A) Let $f(x) = 12\ln\left(\frac{13}{x}\right)$. Find $f'(x)$. Hint: Start by rewriting f using a rule of logarithms.

$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

$f(x) = 12\ln\left(\frac{13}{x}\right) = 12(\ln(13) - \ln(x)) = 12\ln(13) - 12\ln(x)$

So $f'(x) = \underbrace{\frac{d}{dx} 12\ln(13)}_{\text{constant}} - \frac{d}{dx} 12\ln(x)$

$= 0 - \frac{12}{x}$ results from yesterday

$\boxed{f'(x) = -\frac{12}{x}}$

(B) Let $f(x) = 12\ln(x^{13})$. Find $f'(x)$. Hint: Start by rewriting f using a rule of logarithms.

$\ln(a^b) = b\ln(a)$

$f(x) = 12\ln(x^{13}) = 12 \cdot 13 \cdot \ln(x)$

$f'(x) = \frac{d}{dx}(12 \cdot 13)\ln(x) = 12 \cdot 13 \cdot \frac{d\ln(x)}{dx} = 12 \cdot 13 \cdot \frac{1}{x}$

$= \frac{12 \cdot 13}{x}$

(C) Let $f(x) = 12x\ln(13)$. Find $f'(x)$.

Start by rewriting $f(x)$ with constants in front.

$f(x) = 12x\ln(13) = 12\ln(13) \cdot x$

$f'(x) = \frac{d}{dx}(12\ln(13)) \cdot x = 12\ln(13) \cdot \frac{d}{dx}x$

$= 12\ln(13) \cdot 1$

$= 12\ln(13)$

Question (D) is on back. ➔

(D)The goal is to find the equation of the line tangent to the graph of the function

$$f(x) = 5 + \ln(x^3)$$

at the point where $x = e^2$.

Remember that the approach is to build the general form of the equation for the tangent line (in point-slope form):

$$(y - f(a)) = f'(a) \cdot (x - a)$$

Part I Get Parts

Identify the number a. $a = e^2$ (this is the x-coord of the point of tangency)

Find $f(a)$. $f(a) = f(e^2) = 5 + \ln((e^2)^3)$

$= 5 + \ln(e^6)$

$= 5 + 6 = 11$

this is the y-coord of the point of tangency.

Find $f'(x)$. Hint: Start by rewriting f using a rule of logarithms.

rewrite $f(x) = 5 + \ln(x^3) = 5 + 3\ln(x)$

then $f'(x) = \frac{d}{dx}5 + \frac{d}{dx}3\ln(x) = 0 + \frac{3}{x} = \frac{3}{x} = f'(x)$

Find $f'(a)$. $f'(a) = f'(e^2) = \frac{3}{e^2}$ = slope m of the tangent line

Part II Substitute Parts Into the Equation

Substitute the parts that you have found into the tangent line equation. Then convert your equation to slope intercept form.

$(y - 11) = \left(\frac{3}{e^2}\right)(x - e^2)$

$y - 11 = \left(\frac{3}{e^2}\right)x - \left(\frac{3}{e^2}\right)e^2$ these cancel

$y - 11 = \left(\frac{3}{e^2}\right)x - 3$

$y = \left(\frac{3}{e^2}\right)x + 8$ this is in the form $y = mx + b$

Section 4-3 The Product Rule + The Quotient Rule

The Product Rule

This Rule is used for taking the derivative of a product of functions.

$$f(x) \cdot g(x)$$

~~Note: The "obvious thing" would be~~

$$\cancel{\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f'(x) \cdot g'(x)}$$

The obvious thing is wrong

The product rule:

$$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Apply this rule

Example (4-3 #13) $f(x) = (-3x^2+5x-7)\cdot(3x-2)$ find $f'(x)$ using Product Rule. Don't simplify.

Solution:

$$f'(x) = \left(\frac{d}{dx} -3x^2+5x-7\right)\cdot(3x-2) + (-3x^2+5x-7)\cdot\left(\frac{d}{dx} 3x-2\right)$$

product rule

$$= (-6x+5+0)\cdot(3x-2) + (-3x^2+5x-7)\cdot(3+0)$$

(Don't simplify)

Example (4-3 #47) $f(x) = (-3x^2+5x-7)\cdot e^{(x)}$. Find $f'(x)$ and simplify.

Solution

$$f'(x) = \left(\frac{d}{dx} -3x^2+5x-7\right)\cdot e^{(x)} + (-3x^2+5x-7)\cdot\left(\frac{d}{dx} e^{(x)}\right)$$

$$= (-6x+5)\cdot e^{(x)} + (-3x^2+5x-7)\cdot(e^{(x)})$$

factor out $e^{(x)}$

$$= (-6x+5-3x^2+5x-7)e^{(x)}$$

$$f'(x) = \left(-3x^2 - x - 2\right)\cdot e^{(x)}$$

Example (4-3#11) $f(x) = 5x^7 \cdot \ln(x)$

(A) Find $f'(x)$ and simplify.

Solution

$$f'(x) = \left(\frac{d}{dx}5x^7\right)\cdot \ln(x) + \left(5x^7\right)\cdot\left(\frac{d}{dx}\ln x\right)$$

$$= \left(35x^6\right)\cdot \ln(x) + \left(5x^7\right)\left(\frac{1}{x}\right)$$

$$= 35x^6 \ln(x) + 5x^6$$

factor out $5x^6$

$$f'(x) = 5x^6\left(7\ln(x) + 1\right)$$

(B) Find $f'(1)$

Solution: $f'(1) = 5\cdot(1)^6\left(7\ln(1) + 1\right)$

$$= 5\cdot 1\;(7\cdot 0 + 1) = 5(1) = 5$$

(c) Find $f'(e)$

Solution $f'(e) = 5 \cdot e^6 (7\ln(e) + 1)$

$= 5e^6(7 \cdot 1 + 1)$

$= 5e^6(7+1)$

$= 5e^6(8)$

$= 40e^6$

The Quotient Rule Used for taking the derivatives of quotients.

functions of the form

$$\frac{top(x)}{bottom(x)}$$

The "obvious thing" would be

$$\cancel{\frac{d}{dx}\frac{top(x)}{bottom(x)} = \frac{top'(x)}{bottom'(x)}}$$

The "obvious thing" is wrong!

The quotient Rule

$$\frac{d}{dx}\left(\frac{top(x)}{bottom(x)}\right) = \frac{top'(x)\cdot bottom(x) - top(x)\cdot bottom'(x)}{(bottom(x))^2}$$

Example 4-3 #17

$$f(x) = \frac{5x^2+7}{2x-3}$$ find $f'(x)$ and simplify

Solution $$f'(x) = \frac{\left(\frac{d}{dx}5x^2+7\right)\cdot(2x-3) - (5x^2+7)\left(\frac{d}{dx}2x-3\right)}{(2x-3)^2}$$

$$= \frac{(10x)\cdot(2x-3) - (5x^2+7)\cdot(2)}{(2x-3)^2}$$

combine

$$= \frac{20x^2 - 30x - 10x^2 - 14}{(2x-3)^2}$$

$$= \frac{10x^2 - 30x - 14}{(2x-3)^2}$$

Example $f(x) = \frac{e^{(x)}}{3x^2-5}$ find $f'(x)$ and simplify.

Solution $f'(x) = \frac{\left(\frac{d}{dx} e^x\right)\cdot(3x^2-5) - e^{(x)}\cdot\left(\frac{d}{dx} 3x^2-5\right)}{(3x^2-5)^2}$

$$= \frac{(e^{(x)})\cdot(3x^2-5) - e^{(x)}(6x)}{(3x^2-5)^2}$$

Simplify by factoring out $e^{(x)}$ in numerator.

$$f'(x) = \frac{e^{(x)}\left(3x^2 - 5 - 6x\right)}{(3x^2-5)^2}$$

Now Do Class Drill 6 in Course Packet

The moral of Class Drill 6

- If you have a product where one of the factors is a constant, you don't need the product rule
- If you have a quotient with a constant denominator, then you don't need the quotient rule.

Use the constant multiple rule in these cases.

Class Drill 6: Don't Forget the Easy Derivative Rules

[1] Let $f(x) = 7(x^2 + 3x + 5)$

(A) Find $f'(x)$, using the Product Rule to deal with the 7 in front.

$$f'(x) = \left(\frac{d}{dx}7\right)\cdot(x^2+3x+5) + 7\left(\frac{d}{dx}x^2+3x+5\right)$$

$$= (0)\cdot(x^2+3x+5) + 7(2x+3)$$

$$= 14x+21$$

(B) Start over. Find $f'(x)$ again, this time using the Constant Multiple Rule to deal with the 7 in front.

$$f'(x) = 7\frac{d}{dx}(x^2+3x+5)$$

$$= 7(2x+3)$$

$$= 14x+21$$

[2] Let $f(x) = \dfrac{(x^2 + 3x + 5)}{7}$

(A) Find $f'(x)$, using the Quotient Rule to deal with the fraction.

$$f'(x) = \frac{\left(\frac{d}{dx}x^2+3x+5\right)\cdot 7 - (x^2+3x+5)\left(\frac{d}{dx}7\right)}{7^2}$$

$$= \frac{(2x+3)\cdot 7 - \cancel{(x^2+3x+5)(0)}}{7^2} \quad \text{zero}$$

$$= \frac{(2x+3)\cdot 7}{7^2} = \frac{(2x+3)}{7}$$

(B) Start over. Find $f'(x)$ again, but this time do not use the Quotient Rule. Instead, start by rewriting f as a constant times a term in parentheses. Then use the Constant Multiple rule

rewrite $f(x) = \dfrac{(x^2+3x+5)}{7} = \left(\dfrac{1}{7}\right)(x^2+3x+5)$

then $f'(x) = \left(\dfrac{1}{7}\right)\left(\dfrac{d}{dx}x^2+3x+5\right) = \left(\dfrac{1}{7}\right)\cdot(2x+3)$