Day 21 is Monday, October 15, 2012

Section 5-1 First Derivatives & Graphs, Continuing.

More terminology

Define "Partition Number"

words: c is a partition number for the function g.

meaning: Either $g(c)=0$ or $g(c)$ DNE.

Use this terminology to rewrite our definition of critical value

Definition of "Critical Value"

Words: c is a critical value for the function ~~g~~ f.

Meaning: Both of the following are satisfied

- $f'(c)=0$ or $f'(c)$ DNE. That is, $x=c$ is partition number for f'.
- $f(c)$ exists

The First Derivative Test

A function f will have a local max or min at some $x=c$ only when the following three conditions are all met.

- $f'(c)=0$ or $f'(c)$ DNE

  that is $x=c$ is partition number for f'

- $f(c)$ exists

  that is $x=c$ is a critical value for f

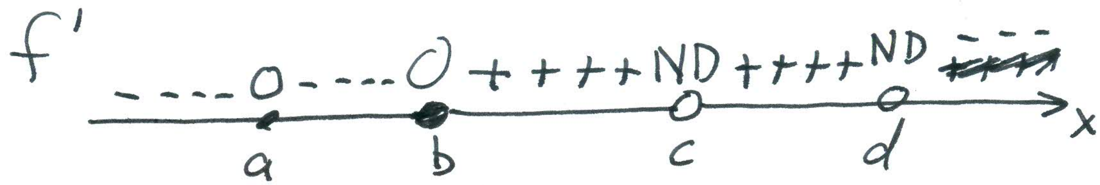
- f' changes sign at $x=c$.

Final example involving graphical approach

exercise 5-1 #10

Given info: f is continuous for all x-values

The sign chart for f' is



Find x-coordinates of maxes

Find x-coordinates of mins.

Solution

- We see that $f'(x)=0$ or $f'(x)$ DNE at $x=a,b,c,d$

  Those are the partition numbers for f'.

- because f is known to be continuous, we know that $f(x)$ exists at every x-value.

  So we know that $x=a,b,c,d$ are all critical values for f.

We see that at the critical value $x=a$,
$f'(x)$ does not change sign.
So f does not have max or min.

at critical value $x=b$, f' changes from neg to pos,
So f ~~has~~ has local min at $x=b$.

at critical value $x=c$, f' does not change sign.
So f' does not have max or min.

at critical value $x=d$, f' changes from pos to neg
So f has local max at $x=d$.

Now analytical examples (function f given by ~~[illegible]~~ formula.)

Example #1

$$f(x) = -x^4 + 50x^2$$

(A) find partition numbers for f'

(B) find critical values for f.

Solution

(A) $f'(x) = \frac{d}{dx}\left(-x^4 + 50x^2\right)$

$= -4x^3 + 100x$

Question: for what x-values is $f'(x) = 0$ or $f'(x)$ DNE?

Observe $f'(x)$ is a polynomial, so $f'(x)$ always exists.

So look for x values that cause $f'(x) = 0$

$$0 = -4x^3 + 100x.$$

$$0 = -4x(x^2 - 25)$$

$$= -4x(x+5)(x-5)$$

Solutions: $x = 0$, $x = -5$, $x = 5$

Those are the <u>partition numbers for f'</u>.

(B) Now find the critical values for f.

Look at the three partition numbers for f'.

At those x-values, does $f(x)$ exist?

Yes $f(x)$ <u>always</u> exits, because f is polynomial

So the $x=0$, $x=-5$, $x=5$ are <u>critical values for f</u>.

Example 2

$$f(x) = 5(x+7)^{2/3}$$

(A) Find partition numbers for f'

(B) Find critical values for f.

Solution

(A) $f'(x) = \frac{d}{dx} 5(x+7)^{2/3}$

$= 5 \cdot \frac{d}{dx} (x+7)^{2/3}$

$= 5 \cdot \frac{2}{3(x+7)^{1/3}} \cdot 1$

$= \frac{10}{3(x+7)^{1/3}}$

Chain rule work

$\text{inner}(x) = x+7$

$\text{inner}'(x) = 1$

$\text{outer}(\ \ ) = (\ \ )^{2/3}$

$\text{outer}'(\ \ ) = \frac{2}{3}(\ \ )^{\frac{2}{3}-1}$

$= \frac{2}{3}(\ \ )^{-\frac{1}{3}}$

$= \frac{2}{3}\frac{1}{(\ \ )^{1/3}}$

$= \frac{2}{3(\ \ )^{1/3}}$

What are the partition numbers for f'?

Since numerator is $\neq 10$ and will <u>never</u> be zero, so $f'(x)$ will <u>never</u> be zero.

But $x=-7$ will cause denominator $=0$, which will cause $f'(-7)$ DNE.

So $x=-7$ is the only partition ~~fo~~ number for f'.

(B) What about <u>critical</u> values for f?

The only candidate is $x=-7$.

So the question is: Does $f(-7)$ exist?

Check $f(-7) = 5(-7+7)^{2/3} = 5\cdot(0)^{2/3} = 0$

$f(-7)$ exists!

So $x=-7$ is a critical value for $f(x)$

Example $f(x) = \dfrac{5}{x+7}$

(A) Find partition numbers for f'

(B) Find critical values for f

Solution

(A) $f'(x) = \dfrac{\left(\cancel{\frac{d}{dx}5}\right)^{0}(x+7) - 5\left(\cancel{\frac{d}{dx}x+7}\right)^{1}}{(x+7)^2} = \dfrac{-5(1)}{(x+7)^2}$

$$f'(x) = \frac{-5}{(x+7)^2}$$

Observe: numerator is -5. It can never be zero.
So $f'(x)$ will never be zero.

But $x=-7$ will cause $f'(-7)$ DNE.

So $x=-7$ is partition number for f'.

(B) Find critical values for $f(x)$.
Our only candidate is $x=-7$. But $f(-7)$ DNE
So $x=-7$ is not a critical value for f.