Day 22 is Tuesday, October 16, 2012

Start with analytical examples involving Section 5-1 concepts.

Example #1 $f(x) = 2x^3 - 3x^2 - 36x$

Find critical values.

Solution $f'(x) = 6x^2 - 6x - 36$

Notice $f'(x)$ is polynomial, so $f'(x)$ always exists.

Are there any x-values that will cause $f'(x) = 0$?

Set $f'(x) = 0$ and solve for x

$$6x^2 - 6x - 36 = 0$$

factor this.

Factor out 6 first

$$6(x^2 - x - 6) = 0$$

$$6(x+2)(x-3) = 0$$

Solutions: $x = -2$, $x = 3$

These are <u>partition</u> numbers for $f'(x)$ (because they cause $f'(x) = 0$)

Are they critical values for $f(x)$?

$f(x)$ <u>exists</u> for <u>every</u> x value.

So any x-value that is a partition number for $f'(x)$ is <u>also</u> a critical value for $f(x)$.

So critical values are $x = -2$, $x = 3$.

Harder example: $f(x) = 4x^3 + 39x^2 - 144x + 2$

Find critical values.

Solution: $f'(x) = 12x^2 + 78x - 144$

How do you factor that?!?

from wolfram: $f'(x)$ factors as $f(x) = 6(x+8)(2x-3)$

So $f'(x)=0$ when $x=-8$ or $x=\frac{3}{2}$

Example #3 $f(x) = -x^4 + 50x^2$

Find intervals where $f(x)$ is increasing or decreasing.

Find the local max + mins (x-values)

Find the y-coordinates at local max + mins.

Solution Study the sign of $f'(x)$.

$f'(x) = -4x^3 + 100x = -4x(x^2-25)$

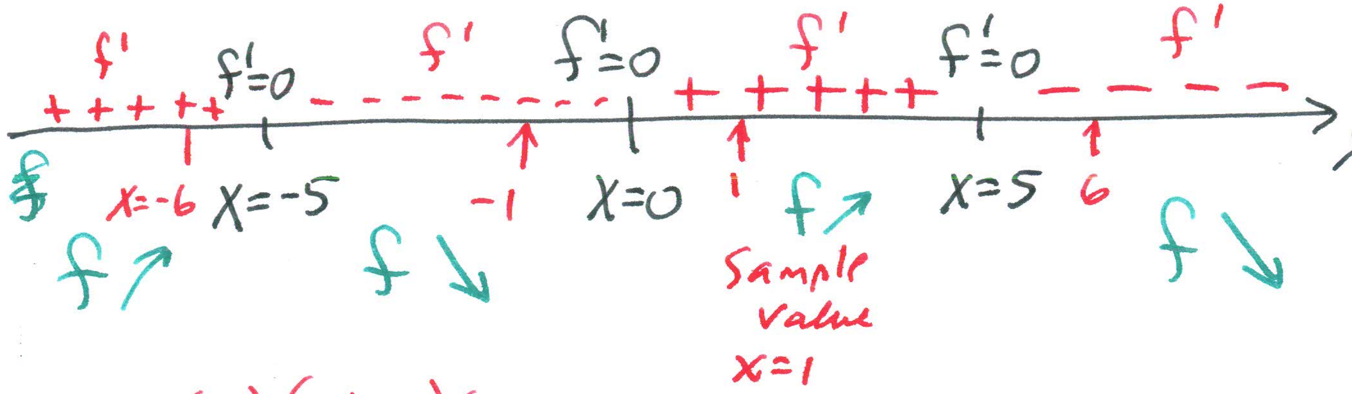
$= -4x(x+5)(x-5)$

critical values $x=0$ $x=-5$ $x=5$

These are the x-values where $f'(x)=0$.

Study sign of $f'(x)$ by making a sign chart.

Sign chart
for $f'(x) = -4x(x+5)(x-5)$



$f'(-6) = -4(-6)(-6+5)(-6-5)$

$= 24(-1)(-11)$

$=$ positive

$f'(-1) = (-4)(-1)(-1+5)(-1-5) = 4(4)(-6) =$ negative

$f'(1) = (-4)(1)(1+5)(1-5) = (-4)(6)(-4) =$ positive

$f'(6) = (-4)(6)(6+5)(6-5) = (-24)(11)(1) =$ negative

Convert (translate) information into information about f.
(use Reference 8) Show translated info in green.

Conclude that

f increasing on the intervals $(-\infty, -5)$ and $(0, 5)$
because f' is positive there.

f decreasing on the intervals $(-5, 0)$ and $(5, \infty)$
because f' is negative there.

~~loca~~
f has local max at $x = -5$ and $x = 5$
because f' changes from pos to neg.
at those critical values.

f has local min at $x = 0$ because
at that critical value, f' changes from
neg to pos.

Now find the y-coordinates of the max & mins.

x-values $x=-5$, $x=0$, $x=5$.

Substitute these x-values into $f(x)$.

(NOT into $f'(x)$, because that would give us the slopes, not the y-values. And we already know that the slopes are 0.)

$$f(-5) = -(-5)^4 + 50(-5)^2$$

$$= -625 + 1250$$

$$= 625$$

So local max at $(x,y)=(-5, 625)$

$$f(0) = -0^4 + 50(0)^2 = 0$$

So local min at $(x,y)=(0,0)$

$$f(5) = -5^4 + 50(5)^2$$
$$= -625 + 1250$$
$$= 625$$

local max at $(x, y) = (5, 625)$

Applied example

A Drug is administered by pill

The drug concentration in the bloodstream is described by the ~~equat~~ function (milligrams per milliliter)

$$c(t) = \frac{0.14t}{t^2+1} \quad \text{for } 0 \le t \le 21$$

(A) find intervals of increase + decrease

(B) find t-coordinate of any local max or min

(C) find $c(t)$ for any local max or min.

Solution Strategy: get $c'(t)$, study its sign behavior.

$$c'(t) = \frac{d}{dt}\left(\frac{.14t}{t^2+1}\right) \qquad \text{quotient rule!}$$

$$= \frac{\left(\frac{d}{dt}.14t\right)(t^2+1) - (.14t)\left(\frac{d}{dt}t^2+1\right)}{(t^2+1)^2}$$

$$= \frac{(.14)(t^2+1) - (.14t)(2t)}{(t^2+1)^2}$$

$$= \frac{.14t^2 + .14 - 2(.14t^2)}{(t^2+1)^2}$$

$$= \frac{-.14t^2 + .14}{(t^2+1)^2}$$

$$= \frac{(-.14)(t^2 - 1)}{(t^2+1)^2}$$ factored out $-.14$

$$c'(t) = \frac{(-.14)(t+1)(t-1)}{(t^2+1)^2}$$ factored $t^2-1 = (t+1)(t-1)$

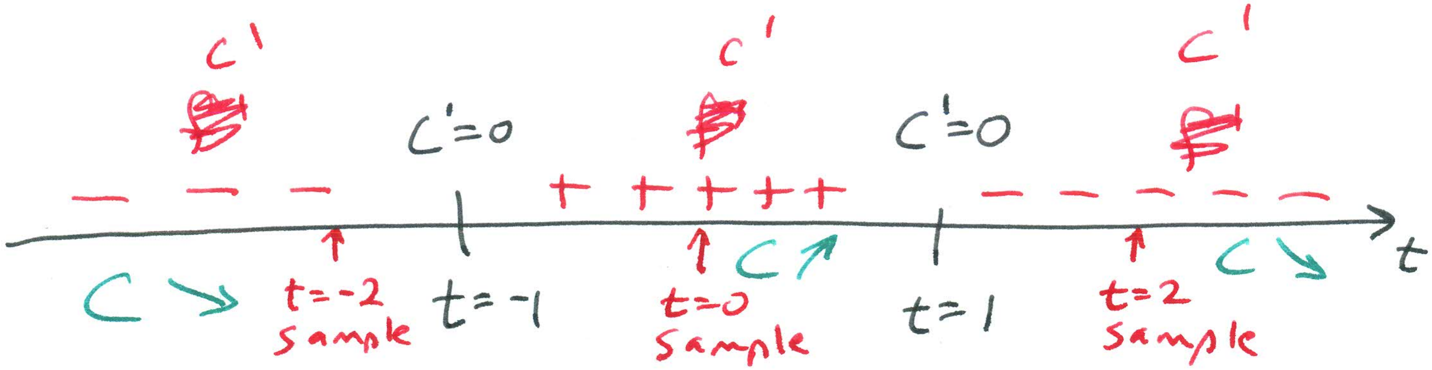
What are the partition numbers for $c'(t)$?
What values of t will cause $c'(t) = 0$
or $c'(t)$ DNE?

Observe denominator is always ≥ 1 so denominator will never be zero. So c' will never be undefined.

Are there any values of t that will cause $c'(t) = 0$

Yes! $t = +1$ and $t = -1$.

Make a sign chart for $c'(t)$



$$c'(0) = \frac{(-.14)(0+1)(0-1)}{(0^2+1)^2} = \frac{(-.14)(1)(-1)}{1} = \text{pos}$$

$$c'(2) = \frac{(-.14)(2+1)(2-1)}{(2^2+1)^2} = \frac{\text{neg}\cdot\text{pos}\cdot\text{pos}}{\text{pos}} = \text{neg}$$

$$c'(-2) = \frac{(-.14)(-2+1)(-2-1)}{(-2^2+1)^2} = \frac{\text{neg}\cdot\text{neg}\cdot\text{neg}}{\text{pos}} = \text{neg}$$

Translate into info about $c(t)$, write in green

Conclusion

$C(t)$ increasing on the interval $(-1,1)$

$C(t)$ decreasing on the intervals $(-\infty,-1)$ and $(1,\infty)$

But wait, our domain is $0 \le t \le 21$

$C(t)$ is increasing on interval $(0,1)$

$C(t)$ is decreasing on interval $(1,21)$

(B) Local max at $t=1$

(C) $C(t)$ coordinate of max is

$$C(1) = \frac{.14(1)}{1^2+1} = \frac{.14}{2} = .07$$ milligrams per milliliter.

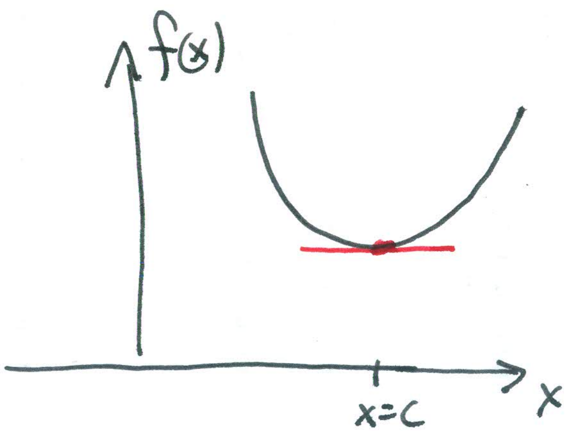
Section 5-2 2nd Derivatives and Concavity

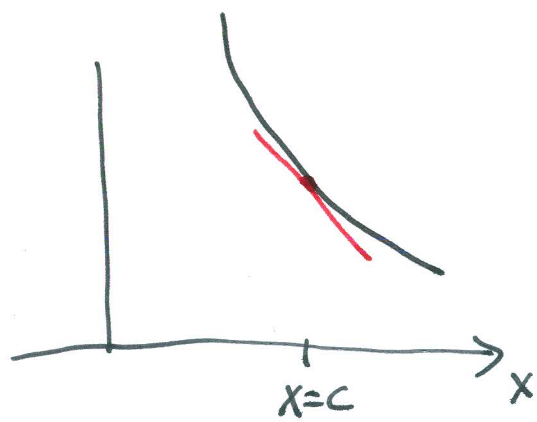
Define Concavity at a particular x-value

words: f is concave up at $x=c$.

meaning: the graph of f has a tangent line at $x=c$ and for x values near $x=c$, the graph of f stays above the tangent line

pictures





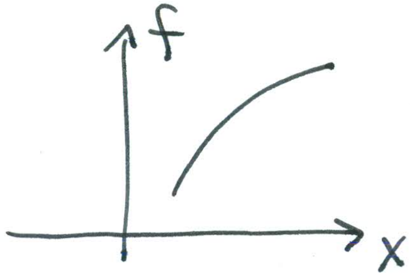
Correspondence

If $f''(c)$ is positive then f is concave up at $x=c$

If $f''(c)$ is negative then f is concave down at $x=c$.

Drill For each graph, tell whether f, f', f'' are pos or neg.

(a)

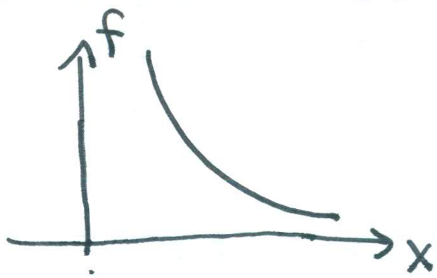


f + because f is above x-axis

f' + because f is increasing

f'' — because f is concave down

(b)



f + f above x-axis

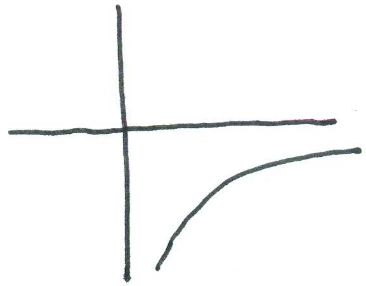
f' − f decreasing

f'' + f concave up

Drill

For each graph, say whether f, f', f''

(c)

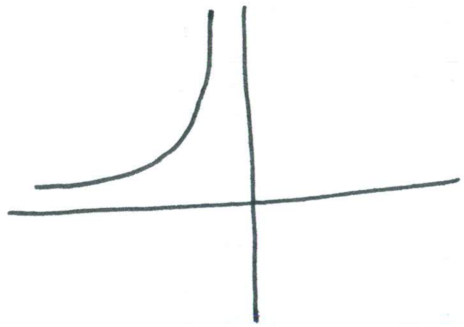


f neg because graph is below x-axis

f' pos because f increasing

f'' neg because f concave down

(d)



f (+) because graph is ~~below~~ above the x-axis

f' (+) because f is increasing

f'' (+) because f is concave up.