Day 24 is Monday, October 22, 2012

Start with Third Analytical Example involving 2nd Derivative + Concavity (Concepts from Section 5-2).

Function $h(x) = e^{\left(-\frac{x^2}{2}\right)} = e^{\left(-\frac{1}{2}\cdot x^2\right)}$

Answer questions (A) - (F) from ~~Friday~~. Thursday.

(A) Find intervals where h is increasing/decreasing/horizontal.

Strategy: find $h'(x)$

Study sign of $h'(x)$.

$$h'(x) = \frac{d}{dx} e^{\left(-\frac{x^2}{2}\right)}$$

$$= outer'(inner(x)) \cdot inner'(x)$$

$$= e^{\left(-\frac{x^2}{2}\right)} \cdot (-x)$$

$$h'(x) = -x \cdot e^{\left(-\frac{x^2}{2}\right)}$$

Chain rule work

$inner(x) = -\frac{x^2}{2}$

$inner'(x) = -\frac{2x}{2} = -x.$

$outer(\;) = e^{(\;)}$

$outer'(\;) = e^{(\;)}$

observe $e^{(\text{anything})}$ is positive, so there are no x-values that will cause $e^{\left(-\frac{x^2}{2}\right)}$ to be zero or undefined.

There is a partition number $x=0$ from the factor x.
So the partition numbers for $h'(x)$ are just $x=0$.

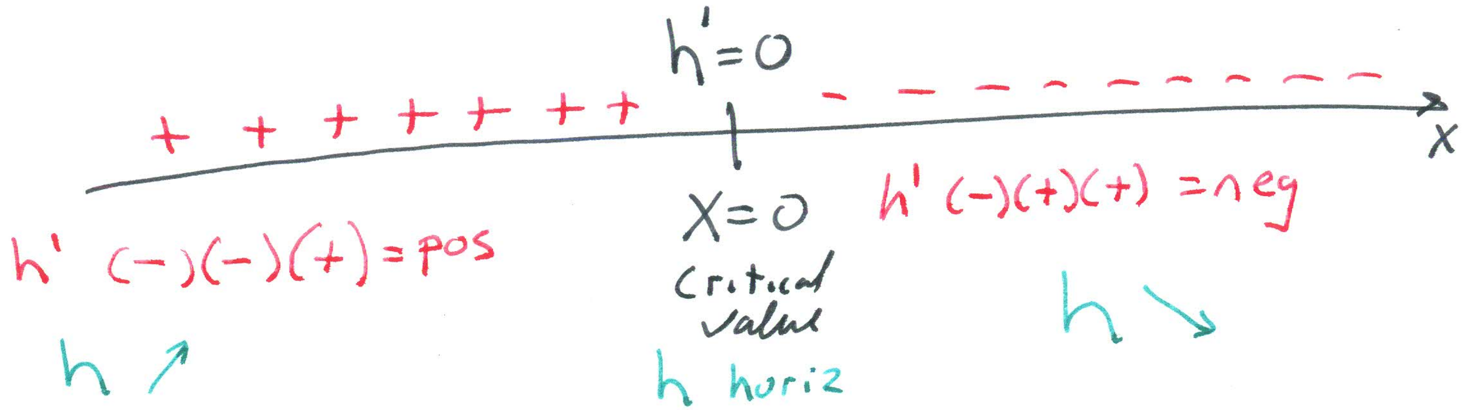
Is that partition number $x=0$ a critical value for $h(x)$?
We have to check to see whether or not $h(0)$ exists.

$$h(0) = e^{\left(-\frac{0^2}{2}\right)} = e^{(0)} = 1$$ this <u>does</u> exist.

So the number $x=0$ is (our only) critical value for h.

Sign chart for $h'(x) = -x \cdot e^{\left(-\frac{x^2}{2}\right)} = \underbrace{(-1)}_{\text{always neg}} \cdot \underbrace{(x)}_{\text{changes sign at } x=0} \cdot \underbrace{e^{(-x^2/2)}}_{\text{always pos}}$

Sign chart for $h'(x) = (-1)\cdot(x)\cdot e^{\left(-x^2/2\right)}$



conclude h increasing on interval $(-\infty, 0)$ because h' pos.

h horizontal at $x=0$ because $h'=0$

h decreasing ~~at~~ on interval $(0,\infty)$ because h' neg.

(B) Find x-coordinates of local max or mins.

Solution $x=0$ is a local max because it is a critical value where h' changes from pos to neg.

(C) Find y-coordinates of local max.

Solution: Earlier, we saw that $h(0) = 1$ y-coord of max

So (x,y) coordinates of max are $(x,y) = (0,1)$.