Day 25 is Tuesday, October 23, 2012

Continuing Section 5-5 Absolute Extrema

The Closed Interval Method

This method is used for
Finding Absolute Max + Absolute Min
in the special situation where
the domain is a closed interval [a, b]
and the function f is known to be
continuous on the domain.

Identify all important x-values (make a list)

- endpoints of the domain
- critical values, that are in the domain.

Then find the corresponding y-values.

The ~~largest~~ greatest y-value on the list is the abs. max.

The smallest y-value on the list is the abs. min.

Example $f(x) = x^4 - 6x^2 + 5$

Find all absolute extrema on the interval $[-3,2]$

Solution

Observe: the domain $[-3,2]$ is a closed interval.

The function f is continuous on the domain (because f is a polynomial)

So we can use the Closed Interval Method.

Identify Important x-values

Endpoints $x = -3$, $x = 2$.

Find Critical Values.

Find partition numbers for $f'(x)$

$f'(x) = 4x^3 - 6(2x) + 0 = 4x^3 - 12x$

we see that f' is polynomial, so f' always exits.

The only possible partition numbers will be from $f'(x) = 0$

So set $f'(x) = 0$ and solve for x.

$$f'(x) = 0$$

$$4x^3 - 12x = 0$$

factor out $4x$

$$4x(x^2 - 3) = 0$$

Solutions: $x = 0$ and $x = \sqrt{3}$ and $x = -\sqrt{3}$

(because $x^2 - 3 = 0$
$x^2 = 3$
$x = \pm\sqrt{3}$)

Those are the partition numbers for $f'(x)$.

We know that $f(x)$ exists at these x-values (because f is polynomial)

So the three numbers $x = -\sqrt{3}$, $x = 0$, $x = \sqrt{3}$ are the critical values for f.

Notice: $\sqrt{3} \approx 1.732$ so all three of these critical values are in the domain $[-3, 2]$

Conclude that the important x-values are

important x		Corresponding y-values
$x=-3$	endpoint	$y=(-3)^4-6(-3)^2+5=81-6\cdot 9+5=81-54+5=32$ max
$x=-\sqrt{3}$	critical	$y=(-\sqrt{3})^4-6(-\sqrt{3})^2+5=-4$ min
$x=0$	critical	$y=0^4-6\cdot 0^2+5=5$
$x=\sqrt{3}$	critical	$y=(\sqrt{3})^4-6(\sqrt{3})^2+5=9-6(3)+5=-4$ min
$x=2$	endpoint	$y=2^4-6(2)^2+5=16-24+5=-3$

Conclude absolute max is $y=32$. (occurs at $x=-3$)

absolute min is $y=-4$ (occurs at $x=-\sqrt{3}$ and $x=\sqrt{3}$)

Related example Same function $f(x) = x^4 - 6x^2 + 5$

Find all absolute extrema on the interval $[-2, 1]$

Solution

~~1.~~ List important x-values

$x = -2$, $x = 1$ endpoints

$x = -\sqrt{3}$, $x = 0$, $\cancel{x = \sqrt{3}}$ critical values

not in domain!

important x-values	corresponding y-values
$x = -2$ endpoint	$y = (-2)^4 - 6(-2)^2 + 5 = 16 - 24 + 5 = -3$
$x = -\sqrt{3}$ critical	$y = -4$ min (from prev. example)
$x = 0$ critical	$y = 5$ max (from prev. example)
$x = 1$ endpoint	$y = 1^4 - 6 \cdot 1^2 + 5 = 1 - 6 + 5 = 0$

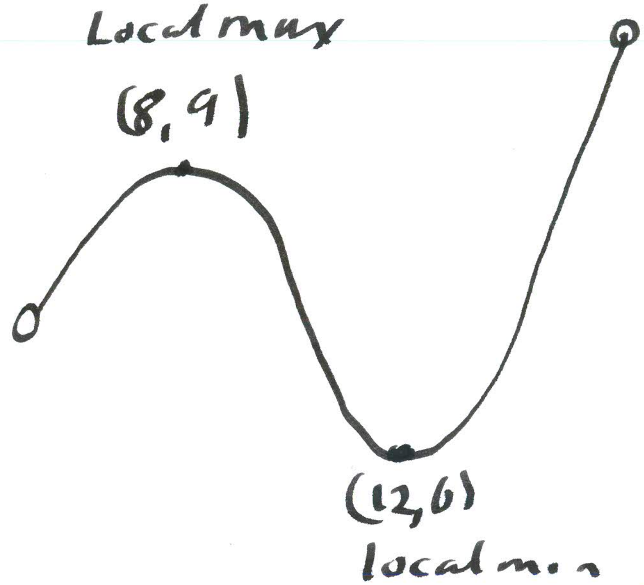
Conclude Absolute max $y = 5$ (occurs at $x = 0$)

Absolute min $y = -4$ (occurs at $x = -\sqrt{3}$)

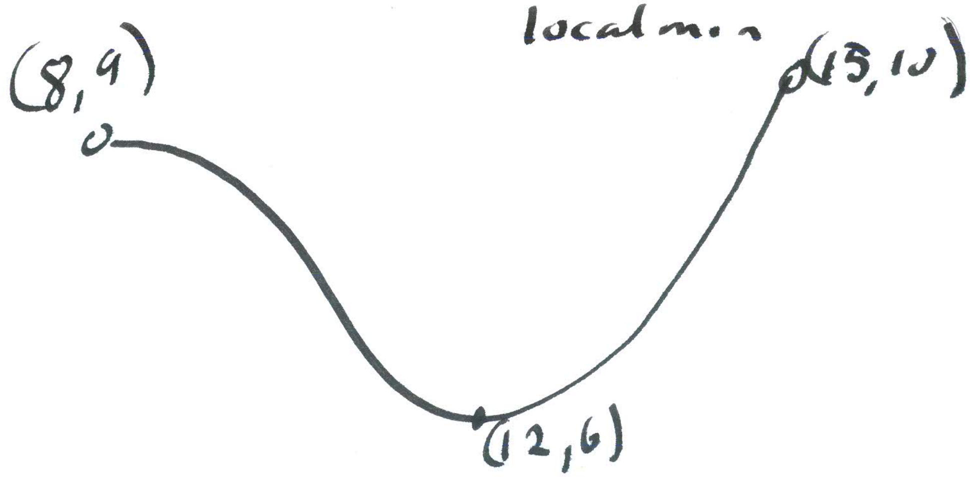
What happens if the domain is <u>not</u> a closed interval, or if the function f is <u>not</u> continuous? Will there be absolute max, min?

<u>Class Drill 12</u>

interval $(6, 15)$



interval $(8, 15)$



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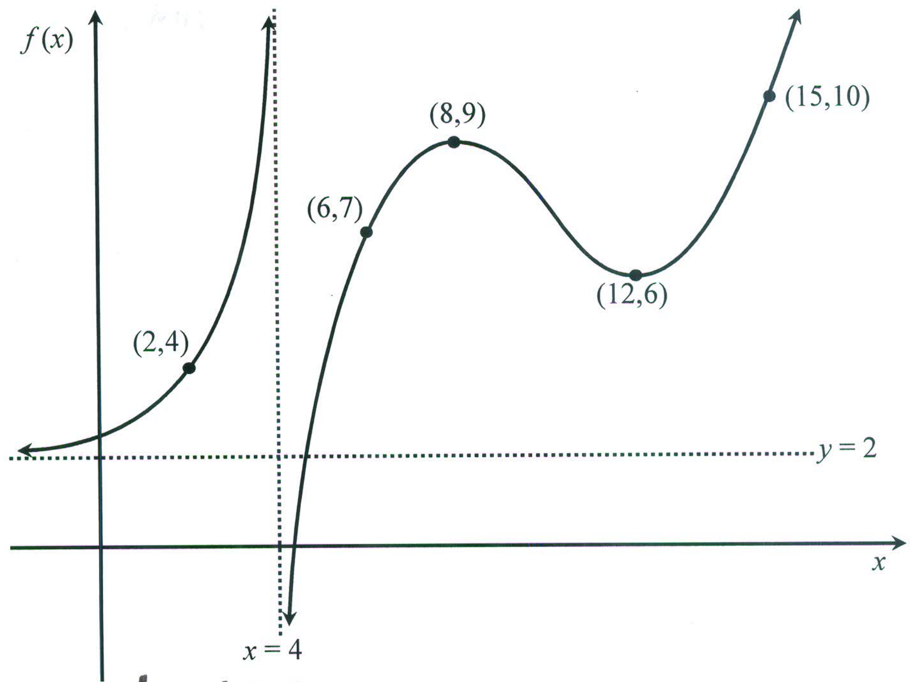
Class Drill 12: Relative and Absolute Extrema

The *Extreme Value Theorem* says that if a function f is continuous on a closed interval $[a,b]$, then f will have both an absolute maximum and an absolute minimum on that interval. In this drill, you investigate what can happen when f is not continuous or the interval is not closed.

The graph of a function f is shown at right. Fill in the table below.



Local max — Local Min

Interval	Relative Maxima in that interval	Relative Minima in that interval	Absolute Max in that interval	Absolute Min in that interval
[6,15]	(x,y)=(8,9) (x,y)=(15,10)	(x,y)=(12,6) (x,y)=(6,7)	y = 10	y=6
(6,15)	(x,y)=(8,9)	(x,y)=(12,6)	none	y=6
(8,15)	none	(x,y)=(12,6)	none	y = 6
[2,12]	(x,y)=(8,9)	(x,y)=(2,4) (x,y)=(12,6)	none	none
(2,12)				
(4,∞)				

Analytical Example Where Extreme Value Theorem Cannot be used.

Let $f(x) = x^4 - 6x^2 + 5$

Find all absolute extrema on the interval $(-\infty, \infty)$.

Observe:

f is even-degree polynomial, with positive leading coefficient, so left + right ends of graph go up. So no absolute max.

Will there be an absolute min? Since f is continuous, it may wiggle around, but there will be no jumps or asymptotes or holes.

So there will be an absolute min.

But where do we look??

Theorem 2 Still holds. Absolute min can only occur at x-values that are endpoints or critical values.

We don't have any endpoints, but we found in earlier example that there were 3 critical values. The absolute min must be at one of these.

recall the values	y-values
$x=-\sqrt{3}$ critical	$y=-4$ min
$x=0$ critical	$y=5$
$x=\sqrt{3}$ critical	$y=-4$ min

Conclude absolute min $y=-4$ (occurs at $x=-\sqrt{3}$, $x=\sqrt{3}$)

remember no absolute max

Harder analytic example

$$f(x) = 20 - 4x - \frac{250}{x^2}$$

Find absolute extrema on the interval $(0,\infty)$

Solution

Observe interval is not closed.

Unfamiliar function. So we can't tell if there will even be an absolute max or min.

But Theorem 2 tells us that if there is a max or min, it will have to be at the critical values in the interval.

Strategy (for Thursday)
- Find Critical Values
- Try to figure out something else.