Day 26 is Thursday, October 25, 2012

Resume Example from Tuesday

$$f(x) = 20 - 4x - \frac{250}{x^2}$$

Find absolute extrema on the interval $(0, \infty)$

Solution

Start by finding the critical values.

Start by rewriting $f(x) = 20 - 4x - 250x^{-2}$

$$f'(x) = \frac{d}{dx}\left(20 - 4x - 250x^{-2}\right)$$

$$= 0 - 4 - 250(-2)x^{-3}$$

$$= -4 + 500\frac{1}{x^3}$$

rewrite without negative exponent.

$$f'(x) = -4 + \frac{500}{x^3}$$

Look for ~~critical values~~ partition numbers for f'.

x-values that cause $f'(x)$ to not exist:

$x=0$ causes $f'(x)$ DNE because cannot divide by zero. So $x=0$ is a partition number for f'.

x-values that cause $f'(x)=0$

$$0 = f'(x)$$

$$0 = -4 + \frac{500}{x^3}$$

add 4 to each side

$$4 = \frac{500}{x^3}$$

multiply both sides by x^3

$$4x^3 = 500$$

divide by 4

$$x^3 = 125$$

$$x = 5$$

So $x=0$ and $x=5$ are the partition numbers for f'.

But are they critical values for f?
Answer by seeing if $f(x)$ exists.

$$f(0) = 20 - 4(0) - \frac{250}{(0^2)} \quad \text{DNE}$$

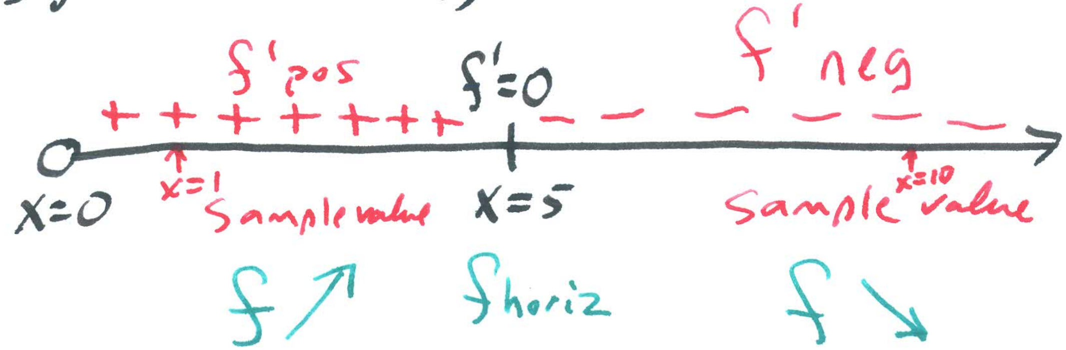
So $x=0$ is not a critical value for f.

$$f(5) = 20 - 4(5) - \frac{250}{5^2} = 20 - 20 - \frac{250}{25} = -10$$

Since $f'(5) = 0$ and $f(5)$ exists, we conclude that $x = 5$ is a critical value.

Next idea: Study the sign behavior of $f'(x)$ to determine more about the critical value $x=5$.

Sign chart for $f'(x)$



$f'(1) = -4 + \frac{500}{(1)^3} = -4 + 500 = \text{pos}$

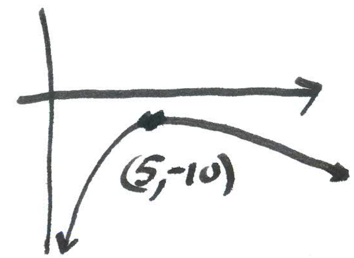
$f'(10) = -4 + \frac{500}{(10)^3} = -4 + \frac{500}{1000} = -4 + \frac{1}{2} = \text{neg}$

Translate into information about f. (in green)

Conclude that $x=5$ is the location of the absolute max.

That is, $y=-10$ is the absolute max.

No abs. min!! This agrees with computer graph



There is *another* way of investigating the critical value $x=5$. Consider the *concavity* of f. Do this by investigating the *sign* of $f''(x)$.

$$f'(x) = -4 + 500x^{-3}$$

$$f''(x) = 0 + 500(-3)(x^{-4}) = -1500 \cdot \frac{1}{x^4}$$

$$f''(x) = -\frac{1500}{x^4}$$

Observe that when $x \neq 0$, x^4 will be positive, so $f''(x)$ will always be negative.

This tells us that f is always concave down

critical value at $x=5$

This shape tells us that $x=5$ will be the location of the absolute max.

This notion of considering concavity of f at a critical value (by studying sign of f'') is what is called the "Second Derivative Test".

Section 5-6 Optimization

Optimization problems are simply absolute max & absolute min problems.

But there may be complications, including

- May be presented as a word problem.
- may have a domain that is not a closed interval.
- the domain might not even be given clearly. You may have to figure out what the domain.
- the function might not be given clearly. You may have to figure out what the function is.
- The function might have more than one variable.
- You might get arrested on Saturday and not be able to study.

Example (Similar to 5-6 #13)

A company manufactures & sells x cameras per week. The weekly price-demand and cost equations are:

$$p = 300 - \frac{x}{30}$$ price-demand equation

$$C(x) = 90{,}000 + 30x$$ Cost function

(A) If the goal is to maximize weekly Revenue, how many cameras should be made per week, and what should be the selling price?

Solution

$$\text{Revenue} = \begin{pmatrix}\text{number of}\\ \text{items made}\end{pmatrix} \cdot \begin{pmatrix}\text{selling price}\\ \text{per item}\end{pmatrix}$$

$$= \text{demand} \cdot \text{price}$$

$$R = x \cdot p$$

$$R = x\left(300 - \frac{x}{30}\right)$$

Notice that this gives us R as a function of x.

$$R(x) = x\left(300 - \frac{x}{30}\right)$$ function notation.

this function can be written a couple of different ways.

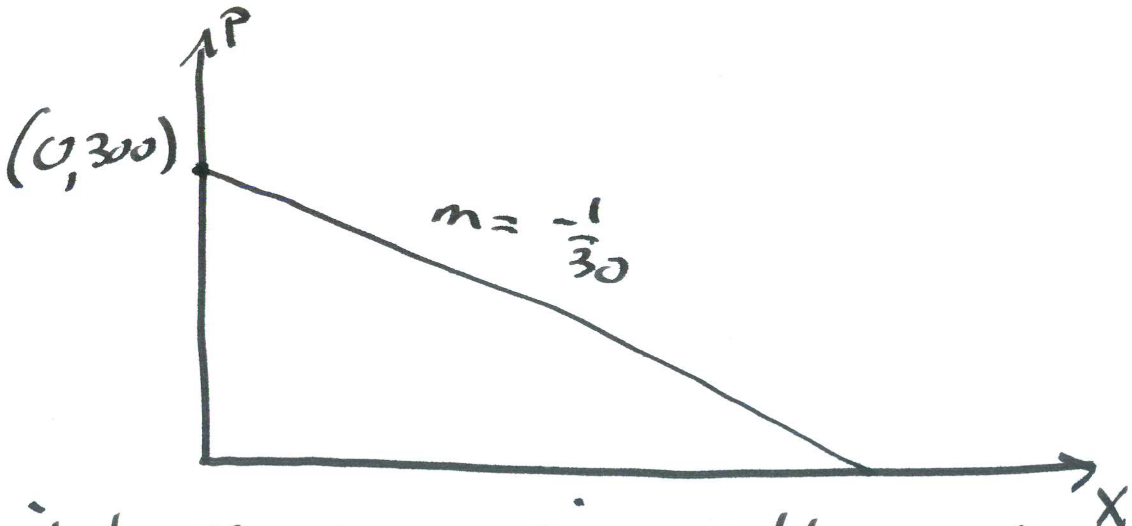
$$R(x) = 300x - \frac{x^2}{30} = \left(-\frac{1}{30}\right)\cdot x^2 + 300x$$

What is the domain?

Must have $x \geq 0$ (can't make negative number of cameras!)

Is there an upper limit on x?

$$p = 300 - \frac{x}{30} = \left(-\frac{1}{30}\right)x + 300$$



Can't have a negative selling price.

What value of x corresponds to $p=0$?

Solve set $p=0$ and solve for x

$$0 = \left(-\frac{1}{30}\right)x + 300$$

$$\left(\frac{1}{30}\right)x = 300$$

$$x = (30)(300) = 9000$$ cameras per week

So domain is $0 \le x \le 9000$

The shape of $R(x)$ will be a parabola facing down. So there should be exactly one critical value.

Find the critical value.

$$R(x) = \left(-\frac{1}{30}\right)x^2 + 300x$$

$$R'(x) = \left(-\frac{1}{30}\right)2x + 300$$

$$= \left(-\frac{1}{15}\right)x + 300$$

What value of x causes $R'(x)=0$?

Set $R'(x)=0$ and solve for x

$$\left(-\frac{1}{15}\right)x + 300 = 0$$

add $\left(\frac{1}{15}\right)x$ to both sides

$$300 = \left(\frac{1}{15}\right)x$$

$$300(15) = x$$

$$4500 = x$$

This number is a partition number for $R'(x)$ and it is a critical value for $R(x)$.

List of important x-values	Corresponding $R(x)$ values
$x=0$ endpoint	$R(0)=0$
$x=4500$ critical value	$R(4500)=675{,}000$ max
$x=9000$ endpoint	$R(9000)=0$

Conclusion To maximize Revenue, make $x = 4500$ cameras per week. The selling price should be

$$p = 300 - \left(\frac{4500}{30}\right) = 300 - 150 = 150 \text{ dollars per camera}$$

the max revenue will be

$$R(4500) = 675{,}000 \text{ dollars per week.}$$