Day 27 is Monday, October 29, 2012

Discuss Quiz #7 ~~Ften.~~ common mistake

$$f'(x) = 3x^2 + 6x - 9$$

You needed to find critical values of f (that is, the partition numbers for f').

Set $f'(x) = 0$ and solve for x

$$3x^2 + 6x - 9 = 0$$

Here are some of the factorizations from student quizzes.

$3(x-2)(x-1)$
$3(x-3)(x+3)$
$3(x-3)(x-1)$
$(3x-9)(x-1)$
$3(x-3)(x+1)$
$3(x+3)(x-1)$

these cannot all be correct.

How can you tell?

Regardless of how you got your factorization

$$f'(x) = 3x^2+6x-9 \xrightarrow{\text{factor}~~\text{ization}~~} 3(x-2)(x-1)$$

Check by multiplying!

$$3(x-2)(x-1) = 3(x^2-x-2x+2) = 3(x^2-3x+2) = 3x^2-9x+6$$

Conclude that the factorization was incorrect!

Try another factorization

$$f'(x) = \cancel{3(x+3)(x-1)}$$

$$3x^2+6x-9 \xrightarrow{\text{factor}} 3(x+3)(x-1)$$

Check by multiplying

$$3(x+3)(x-1) = 3(x^2-x+3x-3) = 3(x^2+2x-3) = 3x^2+6x-9 \checkmark$$

Resume Section 5-6 Optimization

Resume Example involving Cameras

(B) If the goal is to maximize weekly Profit, how many cameras should be made per week, and what should be the selling price?

Solution

We need the profit function $P(x)$

capital P.

$$P(x) = R(x) - C(x)$$

$$= \underbrace{\left(\left(-\tfrac{1}{30}\right)x^2 + 300x\right)}_{\text{from our work on Thursday}} - \underbrace{\left(90{,}000 + 30x\right)}_{\text{given on Thursday}}$$

$$= \left(-\tfrac{1}{30}\right)x^2 + 270x - 90{,}000$$

We need to find the value of x that maximizes $P(x)$.

Note that the domain of $P(x)$ is $0 \le x \le 9{,}000$ because the domain of $R(x)$ was that interval, and $R(x)$ was used in making $P(x)$.

Find critical values of $P(x)$.

find $P'(x)$

Set $P'(x) = 0$

Solve for x.

$$P'(x) = \left(-\frac{1}{30}\right)(2x) + 270 + 0$$

$$= -\frac{x}{15} + 270$$

Set $P'(x) = 0$ and solve for x

$$-\frac{x}{15} + 270 = 0$$

$$270 = \frac{x}{15}$$

$$15(270) = x$$

$x = \cancel{3150}\ 4050$ cameras per week.

Selling price: $p = 300 - \frac{x}{30}$ price-demand equation from Thursday

$$p = 300 - \frac{4050}{30}$$

$p = 165$ dollars per camera

Abstract example

Find positive numbers x, y such that

- The product xy is ~~[illegible]~~ 9000
- The sum $10x + 25y$ is minimized.

Solution

Step 1 Write equation I involving x, y

$$x \cdot y = 9000 \quad \text{equation I}$$

Step 2 Write equation II involving x, y and the letter S for sum.

$$10x + 25y = S \quad \text{equation II.}$$

Step 3 (eliminate one of the variables)

Solve equation I for y.

$$y = \frac{9000}{x}$$

Substitute into equation #2

$$10x + 25\left(\frac{9000}{x}\right) = S$$

Notice that this describes the sum S as a function of one variable, x.

$$S(x) = 10x + \frac{25(9000)}{x}$$ function notation

Step 4 Using calculus find the value of x that minimizes $S(x)$.

Remember: the domain is $x > 0$ (x positive)

Find $S'(x)$

Start by rewriting $S(x) = 10x + 25(9000)x^{-1}$

then $S'(x) = 10 + 25(9000)(-1)x^{-2}$

$$= 10 - \frac{25(9000)}{x^2}$$

Looking for critical values of S.

~~The~~ Start by finding partition numbers for S'.

$x = 0$ because $S(0)$ DNE.

Set $S'(x) = 0$ and solve for x

$$0 = 10 - \frac{25(9000)}{x^2}$$

Solve for x^2

$$\frac{25(9000)}{x^2} = 10$$

$$25(9000) = 10x^2$$

$$\frac{25(9000)}{10} = x^2$$

$$25(900) = x^2$$

$$\sqrt{25(900)} = x$$

$$\sqrt{25} \cdot \sqrt{900} = x$$

$$5 \cdot 30 = x$$

$$x = 150$$

We have found a partition number for S'.
So it is a critical value for S.

How do we determine if it is a max
or a min or neither?

See if S'' is pos or negative.

$$S'(x) = 10 - 25(9000)x^{-2}$$

$$S''(x) = 0 - 25(9000)(-2)(x^{-3})$$

$$S''(x) = \frac{25(9000)(2)}{x^3}$$

at our critical value $x = 150$, we see
that $S''(150) = \frac{25(9000)(2)}{(150)^3}$ = <u>positive</u>!

This tells us that graph of S is concave <u>up</u>.

So $x = 150$ is a min

So $x=150$ is the best value for x.

The corresponding value of y?

Go back to equation I

$$x \cdot y = 9000$$

$$y = \frac{9000}{x}$$

use $x=150$

$$y = \frac{9000}{150}$$

$$y = 60$$

Solution $(x,y) = (150, 60)$