Day 33 is Tuesday, Nov 13, 2012

Start with a couple of examples using Section 6-2 techniques.

Example similar to 6-2#75 (Biology)

A yeast culture weighs 1gm initially and is growing at a rate of

$$W'(t) = .3e^{(.1t)} \text{ gm/hour}.$$

(A) Find the weight function $W(t)$

(B) Find the weight of the culture at time $t = 24$ hours.

Solution

(A) We must find the particular antiderivative of $W'(t) = .3e^{(.1t)}$ that satisfies $W(0) = 1$.

Start by getting the general antiderivative.

$$W(t) = \int W'(t)\,dt$$

$$= \int .3e^{(.1t)}\,dt$$

$$= .3\int e^{(.1t)}\,dt$$

integrand: $e^{(.1t)}$ nested

inner(t) = .1t

outer( ) = $e^{(\ )}$

therefore outer( ) = $e^{(\ )}$

First try:

$W(t) = .3 \cdot \text{outer}(\text{inner}(t)) + C$

$= .3 \cdot e^{(.1t)} + C$

Check:

$$W'(t) = \frac{d}{dt}W(t) = \frac{d}{dt}\left(.3e^{(.1t)} + C\right) = .3\frac{d}{dt}e^{(.1t)}$$

Chain rule work
inner(t) = .1t
inner'(t) = .1
outer( ) = $e^{(\ )}$
outer'( ) = $e^{(\ )}$

$W'(t) = (.3)e^{(.1t)} \cdot (.1)$

This is not what we wanted! We wanted $W'(t) = .3e^{.1t}$

We are off by a factor of .1.

So divide old W(t) by .1 to get a new W(t) and try again

2nd try $W(t) = \frac{.3e^{.1t}}{(.1)} + C = 3e^{.1t} + C$

Check

$W'(t) = \frac{d}{dt}W(t) = \frac{d}{dt}(3e^{(.1t)} + C) = 3\frac{d}{dt}e^{(.1t)}$

$= 3e^{(.1t)} \cdot (.1) = .3e^{.1t}$ ✓