Day 35 is Monday, Nov 19, 2012

Please sit in groups of 3.

Section 6-5 The Fundamental Theorem of Calculus

We have seen two uses of the $\int$ symbol.

① The Indefinite Integral $\int f(x)\,dx = F(x) + C.$

where $F(x)$ is an antiderivative of $f(x)$.
That is, $F'(x) = f(x)$.

② The Definite Integral

$$\int_{x=a}^{x=b} f(x)\,dx = A = \text{"Signed area" between the graph of } f \text{ and the x-axis from } x=a \text{ to } x=b.$$

When doing definite integrals, sometimes we could actually compute the ~~area A~~ signed area A, if the region was made up of simple geometric shapes,

In more difficult examples, where the region was not made up of simple shapes, we found approximations using Riemann sums.

Example $A \approx L_5$

Sometimes, in cases of functions that were increasing, we could ~~sa~~ sandwich the unknown value A in between two approximate values,

For example

$$L_{100} < A = \int_{x=a}^{x=b} f(x)\,dx < R_{100}$$

In Fact, we defined

$$A = \int_{x=a}^{x=b} f(x)\,dx \overset{\text{definition}}{=} \lim_{n\to\infty} S_n = \lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n$$

So far, it seems strange that

1. The Indefinite Integral $\int f(x)\,dx$

2. The Definite Integral $\int_{x=a}^{x=b} f(x)\,dx$

use such similar-looking symbols. They seem to be unrelated concepts.

Here is the relationship between the two concepts:

The Fundamental Theorem of Calculus

$$\int_{x=a}^{x=b} f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is an antiderivative of $f(x)$, obtained by the indefinite integral $\int f(x)\,dx = F(x) + C$.

Example 6-5#10 (A) Find $\int_2^{10} 2x-1\,dx$, using Fundamental Theorem

Solution

(A) Find $\int_{x=2}^{x=10} 2x-1\,dx$ using Fundamental Theorem.

Start by identifying $f(x)=2x-1$, $a=2$ $b=10$

Then use the antiderivative formulas to find

$$F(x) = \int f(x)\,dx = \int 2x-1\,dx$$

$$= 2\int x\,dx - \int 1\,dx$$

$x = x^1$ $n=1$ $\qquad$ $1 = x^0$ $n=0$

$$= 2\cdot\left(\frac{x^{1+1}}{1+1}\right) - \left(\frac{x^{0+1}}{0+1}\right) + C$$

$$= \frac{2x^2}{2} - \frac{x}{1} + C$$

$$F(x) = x^2 - x + C$$

Check to see if $F'(x) = f(x)$.

$$F'(x) = \frac{d}{dx}\left(x^2 - x + C\right) = 2x - 1 + 0$$

$$= 2x - 1$$

$$F'(x) = f(x)$$

So we have a good antiderivative

$$F(x) = x^2 - x + C$$

Now Find $F(b) = F(10) = 10^2 - 10 + C = 100 - 10 + C$

$$F(b) = 90 + C$$

$$F(a) = F(2) = 2^2 - 2 + C = 4 - 2 + C = 2 + C$$

So $F(b) - F(a) = F(10) - F(2) = (90 + \cancel{C}) - (2 + \cancel{C}) = 88$

the C cancels

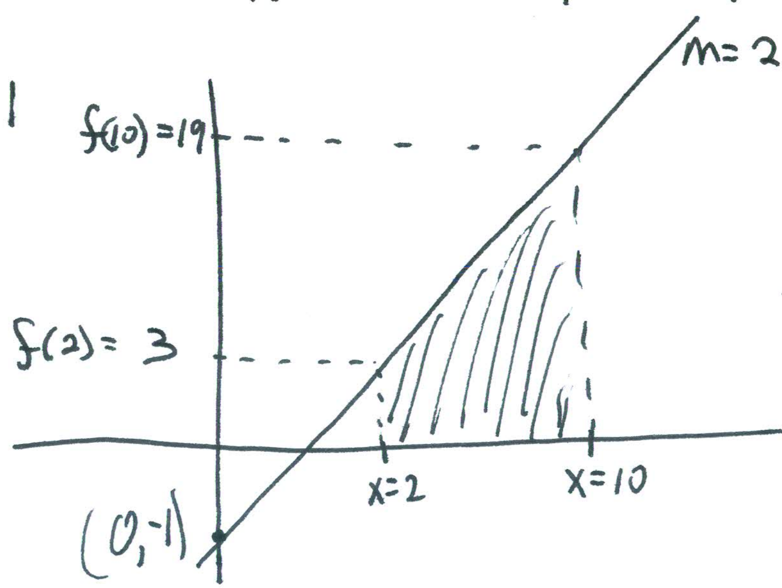
Conclusion $\int_{x=2}^{x=10} 2x-1\,dx = 88$

(B) Find $A=\int_{x=2}^{10} 2x-1\,dx$ geometrically.

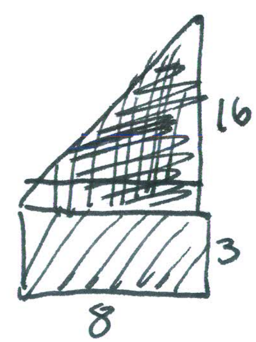
Solution: A is the signed area ~~under~~ between graph of $f(x)$ and the x axis from $x=2$ to $x=10$.

Graph $f(x)=2x-1$

$A =$



$=$



$$= 8\times 3 + \frac{1}{2}(8)(16) = 24 + 8\cdot 8 = 24 + 64 = 88$$

Observe that the results of (A), (B) match

(A) $F(b) - F(a) = 88$

(B) Area $A = 88$

That's what the Fundamental Theorem Says

$$\text{Area } A = \int_{x=a}^{x=b} f(x)\,dx = F(b) - F(a)$$

<u>Now Do New Class Drill</u> Finding Signed Area Under Graph

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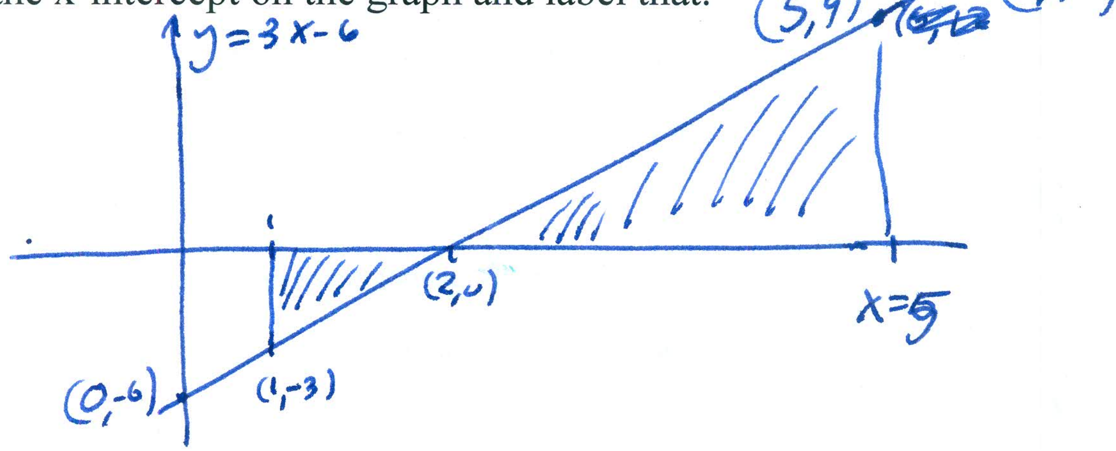
LAST NAME	,	FIRST NAME

Finding the Signed Area Under a Graph By Two Different Methods

2012 - 2013 Fall Semester MATH 1350 (Barsamian) Class Drill
(using concepts from Sections 6-4 and 6-5)

Let $f(x) = 3x - 6$.

(a) Draw the graph of f for $0 \le x \le 6$. Make your graph large and neat. Find the coordinates of the x-intercept on the graph and label that.



(b) On your graph, shade the region between the graph of f and the x-axis from $x = 1$ to $x = 5$.

(c) Find the area of the shaded region. (The unsigned area.) It should be the sum of the positive numbers that are the areas of the two triangles.

Unsigned area =  + 

$$= \frac{1}{2}(1)(3) + \frac{1}{2}(3)(9) = \frac{3}{2} + \frac{27}{2} = \frac{30}{2} = 15$$

(d) Using the known areas of the two triangles, find the signed area of the shaded region. That is, using geometry, find the value of

$$A = \int_{x=1}^{x=5} f(x)dx = \int_{x=1}^{x=5} 3x - 6dx$$

$$A = -\frac{3}{2} + \frac{27}{2} = \frac{+24}{2} = 12$$

(e) Use the antiderivative formulas to find an antiderivative $F(x)$ for $f(x)$. That is, use the antiderivative formulas to find

$$F(x) = \int f(x)dx = \int 3x - 6dx$$

$F(x) = \frac{3}{2}x^2 - 6x + C$

(f) Check: Does $f'(x) = f(x)$? If not, then go back to step (e) and check your work.

Check $F'(x) = \frac{d}{dx}\left(F(x)\right) = \frac{d}{dx}\left(\frac{3}{2}x^2 - 6x + c\right)$

$= \frac{3}{2}(2x) - 6x + 0 = 3x - 6 = f(x)$ ✓

(g) Using the function $F(x)$ that you found, compute $F(5) - F(1)$.

$F(5) = \frac{3}{2}(5)^2 - 6(5) + c = \frac{3}{2}(25) - 30 + c = 7.5 + c$

$F(1) = \frac{3}{2}(1)^2 - 6(1) + c = \cancel{\ } - 4.5 + c$

$F(5) - F(1) = (7.5 + c) - (-4.5 + c) = 12$

(h) Does your answer to question (d) match your answer to question (g)? That is, does $A = F(5) - F(1)$? That is, is the following equation true?

$$\int_{x=1}^{x=5} 3x - 6dx = F(5) - F(1)$$

yes $\quad 12 = 12$

part(d) $\quad$ part(e)

They match!