Day 36 is Tuesday, November 20, 2012

Today: More Examples Using the Fundamental Theorem of Calculus (from Section 6-5)

Example #1 (exercise 6-5 #12) Find $A = \int_{x=0}^{x=2} 4e^{(x)}\,dx$

Solution

Strategy: Use the Fundamental theorem

$$A = \int_{x=0}^{x=2} 4e^{(x)}\,dx = F(2) - F(0)$$

Steps

Find $F(x) = \int 4e^{(x)}\,dx$

Check $F(x)$

$\left\{ \begin{array}{l} \text{Find } F(2) \\ \text{Find } F(0) \end{array} \right.$

Find $F(2) - F(0)$ this will be the value of A.

Step 1 Find $F(x) = \int 4e^{(x)}dx$ note: $f(x) = 4e^{(x)}$ (the integrand)

$$= 4\int e^{(x)}dx$$

$$= 4e^{(x)} + C$$

Step 2 Check: $F'(x) = \frac{d}{dx}\left(4e^{(x)} + C\right) = 4e^{(x)} = f(x)$ ✓

Step 3 $F(2) = 4e^{(2)} + C$

~~Step 4~~ $F(0) = 4e^{(0)} + C = 4 \cdot 1 + C = 4 + C$

Step 4 $F(2) - F(0) = (4e^2 + C) - (4 + C) = 4e^2 - 4 = 4(e^2 - 1)$

this is a number pretty close to 32

Conclusion $A = 4(e^2 - 1) = \int_{x=0}^{x=2} 4e^{(x)}dx$

Example #2 6-5 #14 Find $A = \int_{x=1}^{x=5} \frac{2}{x}\,dx$ $f(x) = \frac{2}{x}$

Solution Same steps, using Fundamental Theorem

Step 1
Find $F(x) = \int \frac{2}{x}\,dx = 2\int \frac{1}{x}\,dx = 2 \cdot \ln|x| + C$

Step 2
Check $F'(x) = \frac{d}{dx}\left(2\ln|x| + C\right) = 2\frac{d}{dx}\ln|x| = 2 \cdot \frac{1}{x} = \frac{2}{x} = f(x)$ ✓

Step 3
$F(5) = 2\ln|5| + C = 2\ln(5) + C$
$F(1) = 2\ln|1| + C = 2\ln(1) + C$
$= 2 \cdot 0 + C = 0 + C = C$

Step 4
$F(5) - F(1) = (2\ln(5) + C) - (C) = 2\ln(5)$

By the Fundamental theorem of Calculus
$$A = F(5) - F(1) = 2\ln(5)$$

<u>Remark</u> Wolfram site says

$$A = \log(25)$$

But when Wolfram writes "log", they mean natural logarithm, "ln".

$$A = \ln(25)$$

Simplify: $25 = 5^2$

So $A = \ln(25) = \ln(5^2) = 2\ln(5)$

This matches our answer.

Example #3 Similar to 6-5 #15 Find $A = \int_{x=-3}^{x=3} x^3 + 5x \, dx$

Solution

Observe $f(x) = x^3 + 5x$

Step 1

$$F(x) = \int x^3 + 5x \, dx = \int x^{\overset{n=3}{3}} dx + 5 \int x^{\overset{n=1}{}} dx$$

$$= \frac{x^{3+1}}{3+1} + 5\left(\frac{x^{1+1}}{1+1}\right) + C$$

$$= \frac{x^4}{4} + \frac{5x^2}{2} + C$$

Step 2 Check

$$F'(x) = \frac{d}{dx}\left(\frac{x^4}{4} + \frac{5x^2}{2} + C\right) = \left(\frac{1}{4}\right)\frac{d}{dx}x^4 + \left(\frac{5}{2}\right)\frac{d}{dx}x^2 + 0$$

$$= \left(\frac{1}{4}\right)(4x^3) + \left(\frac{5}{2}\right)(2x) = x^3 + 5x = f(x) \checkmark$$

Step 3

$$F(3) = \frac{3^4}{4} + \frac{5(3)^2}{2} + C = \frac{81}{4} + \frac{45}{2} + C$$

$$= \frac{81}{4} + \frac{90}{4} + C = \frac{171}{4} + C$$

$$F(-3) = \frac{(-3)^4}{4} + \frac{5(-3)^2}{2} + C = \frac{81}{4} + \frac{45}{2} + C = \frac{171}{4} + C$$

Step 4

By Fundamental Theorem of Calculus,

$$A = F(3) - F(-3) = \left(\frac{171}{4} + \cancel{C}\right) - \left(\frac{171}{4} + \cancel{C}\right)$$

$$A = 0 \quad ?!?$$

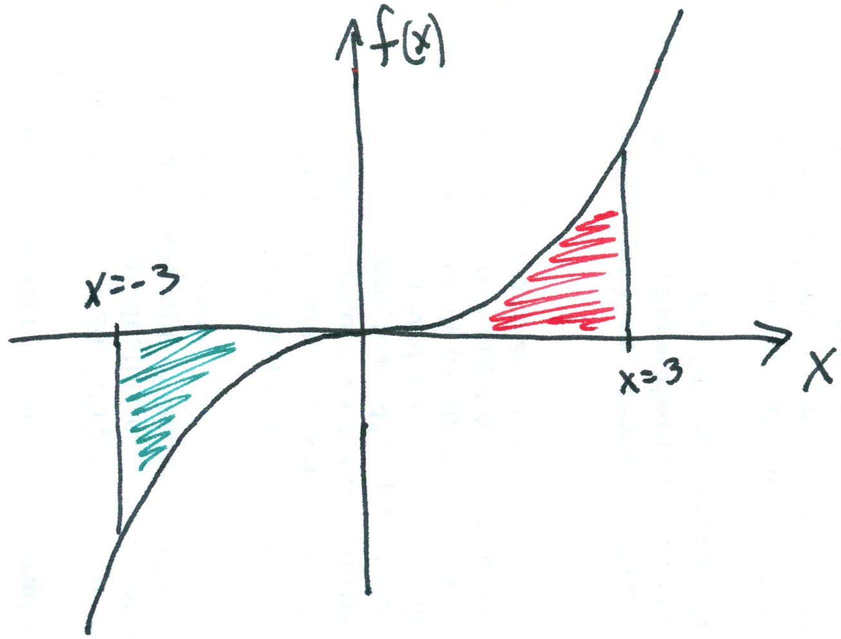
Why does this make sense?

Observe that the integrand $f(x)$ is an "odd function"

$$f(x) = x^3 + 5x = x^3 + 5x^1$$

only have odd powers of x.

Since f(x) is an "odd function", we know that the left side of graph of f is an upside-down mirror image of the right side.



These two areas cancel

A = −green area + red area

= 0