Day 36 is Tuesday, November 20, 2012

Today: More Examples Using the Fundamental Theorem of Calculus (from Section 6-5)

Example #1 (exercise 6-5 #12) Find $A = \int_{x=0}^{x=2} 4e^{(x)}\,dx$

Solution

Strategy: Use the Fundamental theorem

$$A = \int_{x=0}^{x=2} 4e^{(x)}\,dx = F(2) - F(0)$$

Steps

Find $F(x) = \int 4e^{(x)}\,dx$

Check $F(x)$

$\begin{cases} \text{Find } F(2) \\ \text{Find } F(0) \end{cases}$

Find $F(2) - F(0)$ this will be the value of A.

Step 1 Find $F(x) = \int 4e^{(x)}dx$ note: $f(x) = 4e^{(x)}$ (the integrand)

$$= 4\int e^{(x)}dx$$

$$= 4e^{(x)} + C$$

Step 2 Check: $F'(x) = \frac{d}{dx}\left(4e^{(x)} + C\right) = 4e^{(x)} = f(x)$ ✓

Step 3 $F(2) = 4e^{(2)} + C$

~~Step 4~~ $F(0) = 4e^{(0)} + C = 4\cdot 1 + C = 4 + C$

Step 4 $F(2) - F(0) = (4e^2 + C) - (4 + C) = 4e^2 - 4 = 4(e^2 - 1)$

this is a number pretty close to 32

Conclusion $A = 4(e^2 - 1) = \int_{x=0}^{x=2} 4e^{(x)}dx$

Example #2 6-5 #14 Find $A = \int_{x=1}^{x=5} \frac{2}{x}\,dx$ $f(x) = \frac{2}{x}$

Solution Same steps, using Fundamental Theorem

Step 1

Find $F(x) = \int \frac{2}{x}dx = 2\int \frac{1}{x}dx = 2\cdot \ln|x| + C$

Step 2

Check $F'(x) = \frac{d}{dx}\left(2\ln|x| + C\right) = 2\frac{d}{dx}\ln|x| = 2\cdot\frac{1}{x} = \frac{2}{x} = f(x)$ ✓

Step 3

$F(5) = 2\ln|5| + C = 2\ln(5) + C$

$F(1) = 2\ln|1| + C = 2\ln(1) + C$

$= 2\cdot 0 + C = 0 + C = C$

Step 4

$F(5) - F(1) = \left(2\ln(5) + C\right) - \left(C\right) = 2\ln(5)$

By the Fundamental theorem of Calculus

$A = F(5) - F(1) = 2\ln(5)$