Day 40 Monday Dec 3, 2012

Sit in groups of 3 as shown on the screen over there ⟶

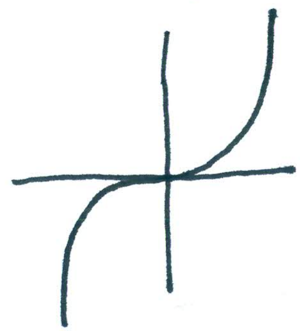
Continue Discussion of Section 7-1

Example Similar to #65 in Section 7-1

Find Area of region bounded by $y = x^5$
$y = 16x$

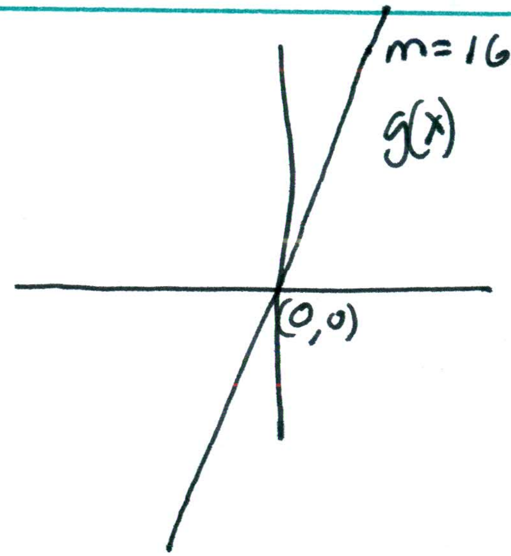
Solution We must figure out how the graphs look.

$f(x) = x^5$



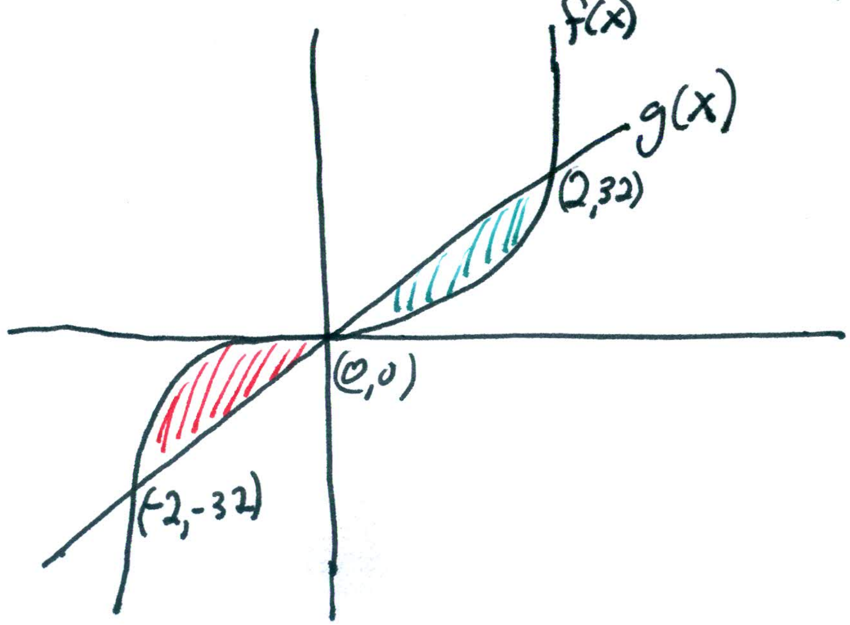
x	y
-3	$(-3)^5 = -243$
-2	$(-2)^5 = -32$
-1	$(-1)^5 = -1$
0	$0^5 = 0$
1	$1^5 = 1$
2	$2^5 = 32$

$g(x) = 16x$



x	y
-3	-48
-2	-32
-1	-16
0	0
1	16
2	32
3	48

Notice that the graphs cross at (-2, -32), (0,0), (2, 32)



Unsigned area (USA)

USA = red area + green area

(f is curve on top) (g is curve on top)

$$= \int_{x=-2}^{x=0} f(x) - g(x)\,dx + \int_{x=0}^{x=2} g(x) - f(x)\,dx$$

$$= \int_{x=-2}^{x=0} x^5 - 16x\,dx + \int_{x=0}^{x=2} 16x - x^5\,dx$$

Pause to introduce notation

The symbol $H(x)\Big|_{x=a}^{x=b}$ means $H(b) - H(a)$

Example $(x^3 - 5x^2 + 11)\Big|_{x=1}^{x=2} = \left(2^3 - 5(2)^2 + 11\right) - \left(1^3 - 5(1)^2 + 11\right)$

$$USA = \left(\frac{x^6}{6} - 8x^2\right)\Bigg|_{x=-2}^{x=0} + \left(8x^2 - \frac{x^6}{6}\right)\Bigg|_{x=0}^{x=2}$$

Check: $\frac{d}{dx}\left(\frac{x^6}{6} - 8x^2\right) = \left(\frac{1}{6}\right)\frac{d}{dx}x^6 - 8\frac{d}{dx}x^2 = \frac{1}{6} \cdot 6x^5 - 8(2x) = x^5 - 16x$ ✓

$$= \cancelto{0}{\left(\frac{0^6}{6} - 8(0)^2\right)} - \left(\frac{(-2)^6}{6} - 8(-2)^2\right) + \left(8(2)^2 - \frac{2^6}{6}\right) - \cancelto{0}{\left(8(0)^2 - \frac{0^6}{6}\right)}$$

$$= -\left(\frac{64}{6} - 32\right) + \left(32 - \frac{64}{6}\right)$$

$$= 32 + 32 - \frac{64}{6} - \frac{64}{6}$$

$$= 64 - \frac{2(64)}{6}$$

$$= 64 - \frac{64}{3}$$

$$= \frac{2(64)}{3} = \frac{128}{3} \approx 42.\overline{66}$$

Compare to ~~Sign~~ the value of

$$\text{Signed Area} = \int_{x=-2}^{x=2} f(x) - g(x)\,dx$$

$$= \int_{x=-2}^{x=2} x^5 - 16x\,dx$$

$$= \left(\frac{x^6}{6} - 8x^2\right)\Big|_{-2}^{2}$$

$$= \left(\frac{2^6}{6} - 8(2)^2\right) - \left(\frac{(-2)^6}{6} - 8(-2)^2\right)$$

$$= \left(\frac{64}{6} - 32\right) - \left(\frac{64}{6} - 32\right)$$

$$= 0$$ Signed area is zero!!

Now do New Class Drill:

Finding the Area Bounded by Curves using <u>Two</u> <u>Different</u> <u>Methods</u>

Front: Method #1

Back: Method #2

LAST NAME , FIRST NAME

Finding the Area Bounded by Curves Using Two Different Methods

2012 - 2013 Fall Semester MATH 1350 (Barsamian) Class Drill

(using concepts from Section 7-1)

The goal is to use two different methods find the (unsigned) of the region bordered by the four lines

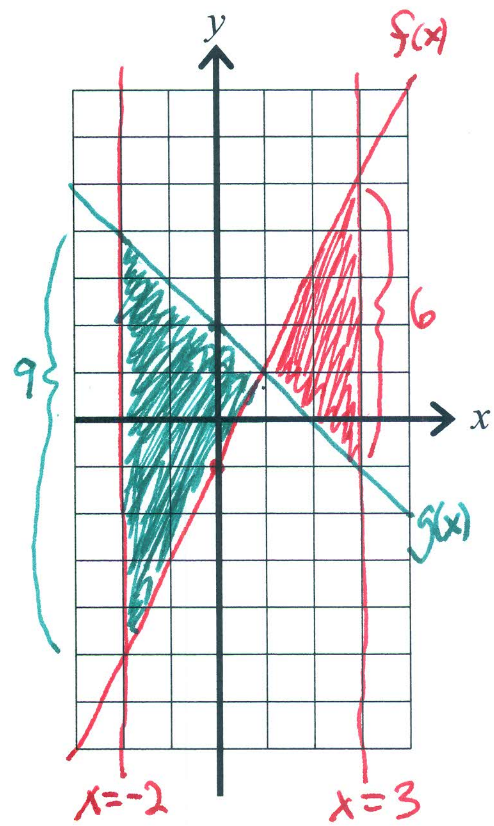
- the line $f(x) = 2x - 1$
- the line $g(x) = -x + 2$
- the line $x = -2$
- the line $x = 3$

Method #1: (Graphical Approach)

Draw the four lines on the graph at right. Be sure to label the lines clearly.

Shade the region bordered by the four lines. (It should be two triangles.)

Find the (unsigned) area of the two triangles.
(Hint: Use the formula $A = \frac{1}{2}bh$ for each triangle.
The formula is easy to use if you choose the base to be the side of the triangle that is vertical.)
Write your results here:



Unsigned Area of Left Triangle = $\frac{1}{2}(9)(3) = \frac{27}{2}$

Unsigned Area of Right Triangle = $\frac{1}{2}(6)(2) = 6$

Now add the two unsigned areas to find the total shaded area:

Unsigned Area = $\frac{27}{2} + 6 = \frac{27}{2} + \frac{12}{2} = \frac{39}{2}$

Method #2 Using Calculus

Set up a sum of definite integrals to compute the (unsigned) area.
Your result should look like this.

$$USA = \int_{x=a}^{x=b} (some\ \ integrand\ here)dx + \int_{x=b}^{x=c} (another\ \ integrand\ here)dx$$

(You will have to figure out the integrands and the limits of integration a, b, c.)
Then use calculus to find the value of the definite integrals and find their sum.

$$USA = \int_{x=-2}^{x=1} g(x) - f(x)\,dx + \int_{x=1}^{x=3} f(x) - g(x)\,dx$$

$$= \int_{x=-2}^{x=1} (-x+2) - (2x-1)\,dx + \int_{x=1}^{x=3} (2x-1) - (-x+2)\,dx$$

$$= \int_{x=-2}^{x=1} -3x + 3\,dx + \int_{x=1}^{x=3} 3x - 3\,dx$$

$$= \left(-\frac{3x^2}{2} + 3x\right)\Bigg|_{x=-2}^{x=1} + \left(\frac{3x^2}{2} - 3x\right)\Bigg|_{x=1}^{x=3}$$

$$= \left(\frac{-3(1)^2}{2} + 3(1)\right) - \left(-\frac{3(-2)^2}{2} + 3(-2)\right) + \left(\frac{3(3)^2}{2} - 3(3)\right) - \left(\frac{3(1)^2}{2} - 3(1)\right)$$

$$= \left(\frac{-3}{2} + 3\right) - \left(-6 - 6\right) + \left(\frac{27}{2} - 9\right) - \left(\frac{3}{2} - 3\right)$$

$$= -\frac{3}{2} + 3 + 12 + \frac{27}{2} - 9 - \frac{3}{2} + 3$$

$$= -3 + 3 + 12 + \frac{27}{2} - 9 + 3$$

$$= 6 + \frac{27}{2} = \frac{39}{2}$$

Same answer!!