Day 42 is Thursday, December 6th, 2012

Continuing Section 7-2

Drawing for example at end of lecture Tuesday

Continuous Income Stream with flow rate

$$f(t) = 600\, e^{(.06t)}$$

(A) Find total income in 1st two years

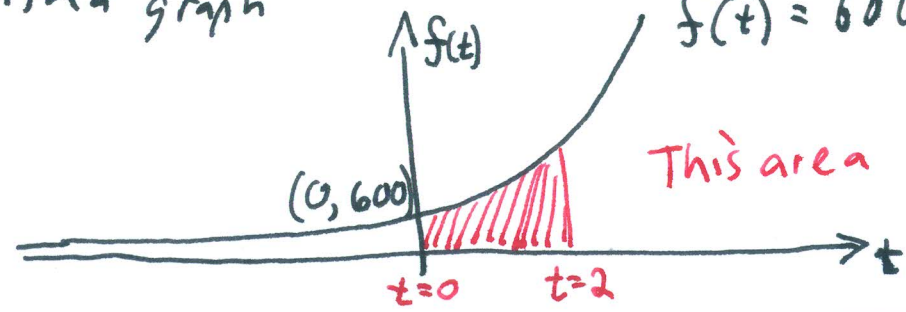
Solution (from Tuesday)

$$\Delta F = F(2) - F(0) = \int_{x=0}^{x=2} f(t)\,dt = \int_{x=0}^{x=2} 600\, e^{(.06t)}\,dt$$

$$= 10{,}000\left(e^{.12} - 1\right) \quad \text{from Tuesday}$$

$$\approx \$1274.97$$

(B) Illustrate with a graph



"Future Value Problems"

Given info about starting balance
and income stream flow rate

and interest rate

Find: the future value of the account
(the balance at some time in future.)

The two "Total Change" problems that we studied on Tuesday could be thought of as simple future value problems.

- zero intial balance
- known flow rate
- accumulated money is not earning any interest.

More difficult Future Value Problems

- may or may not have initial balance
- known & income stream flow rate $f(t)$
- accumulated money is going to be earning interest. (Continuously compounded interest e^{rt}).

Formula for Future Value of a Continuous Income Stream

$$FV = \int_{t=0}^{t=T} f(t)e^{r(T-t)}dt = \int_{t=0}^{t=T} f(t)\,e^{rT}e^{-rt}dt = e^{rT}\int_{t=0}^{t=T} f(t)e^{-rt}dt$$

$e^{a+b} = e^a e^b$

t is time in years (a variable)

T is the future time (in years) when you want to know what the value will be. (T is a constant)

Example Future Value Problem 7-2 #30

At age 25, you start depositing $2000/year into an IRA account. (consider this as a continuous income stream with flow rate $f(t) = \$2000/\text{year}$)

The account earns 3% interest compounded continuously.

What will be the balance when you are 65 years old?

Solution

t = time in years since you turned 25

$T = 40$ years (we are interested in the account value 40 years in the future.)

$r = .03$ (3% interest)

$$FV = e^{(.03)(40)} \int_{t=0}^{t=40} 2000\, e^{-.03t}\, dt$$

$$= e^{(.03)(40)} \cdot 2000 \int_{t=0}^{t=40} e^{(-.03t)}\, dt$$

$$= e^{(.03)(40)} \cdot 2000 \left(\frac{e^{(-.03t)}}{(-.03)} \right) \Bigg|_{t=0}^{t=40}$$

$$= \frac{e^{(.03)(40)}(2000)}{(-.03)} \left[e^{(-.03t)} \right] \Bigg|_{\tau=0}^{t=40}$$

$$= \frac{e^{.03(40)}(2000)}{(-.03)} \left(e^{-.03(40)} - e^{-.03(0)} \right)$$

$$\approx 1.52 \times 10^5 = 1.52 \times 100000$$

$$= \$152{,}000$$

Related question

What is the Present Value of that income stream?

That is, what lump sum ~~deposit~~ initial deposit into an account with the same interest rate would have the same future value?

$$FV = 152{,}000$$

Question becomes this: Using formula for continuously-compounded interest

$$A = Pe^{(rt)}$$

with $A = 152{,}000$

$r = .03$

$t = 40$

what is the value of P?

Remember our approach

Solve the equation $A = Pe^{rt}$ for P

result: $P = \frac{A}{e^{(rt)}}$

Substitute in $A = 152{,}000$, $r = .03$, $t = 40$

$$P = \frac{152{,}000}{e^{(.03)(40)}} \simeq \$45781$$

The Last Topic!!

Consumer Surplus problems

Producer Surplus problems

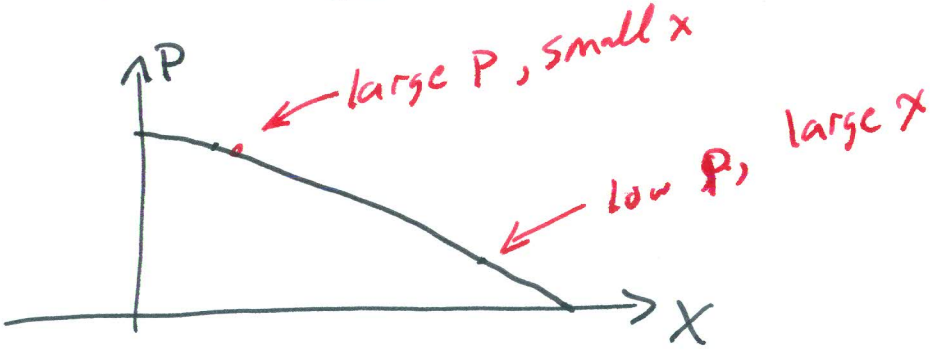
Consumer Surplus Problems

x = demand
= number of items that people are willing to buy

p = selling price per item.

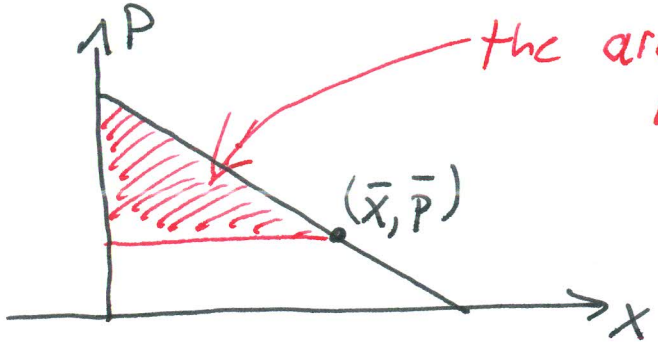
price demand equation relates p, x

P is Usually a decreasing function of x



Define Consumers Surplus

given ~~a~~ known $(\bar{x}, \bar{p})$ pair on the graph.
(label them $(\bar{x}, \bar{p})$)



the area of this shaded region is called the ~~"Cons"~~ "Consumer's surplus"

The idea is that many people would be willing to buy the item at prices higher than $\bar{p}$.

The consumers surplus (CS) is a measure of how much the consumers have saved (overall) if the selling price is $\bar{p}$.

Example 7-2 #44, 46

Price-demand equation ~~$p = 400 - .05x$~~

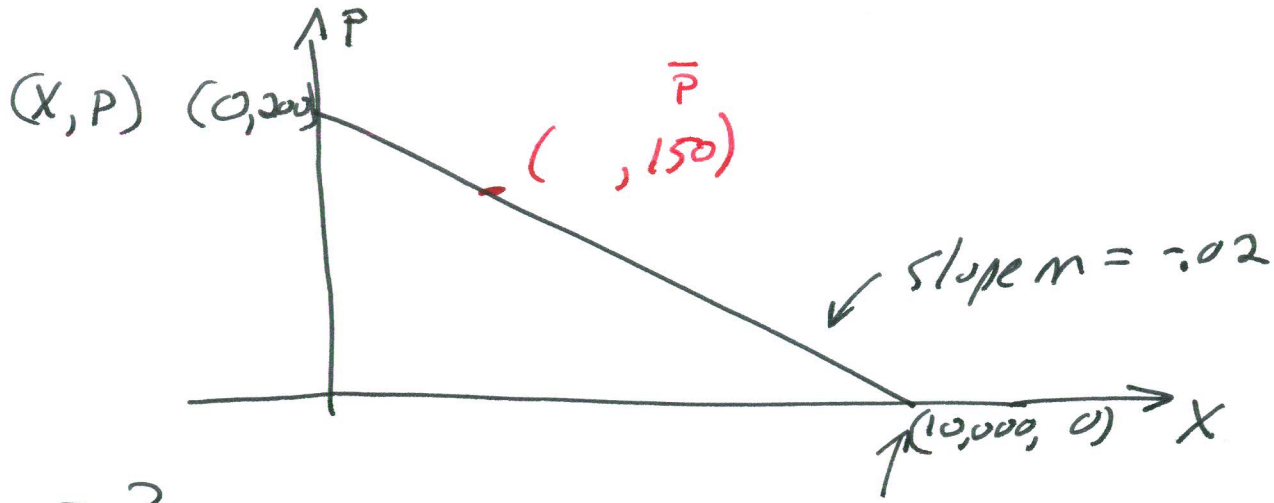
$p = 200 - .02x$

Known price level $\bar{p} = \$150$.

Find the consumer's Surplus.

Solution

Graph the price-demand equation $p = 200 - .02x$



What is $\bar{x}$?

$$p = 200 - .02x$$

$$p - 200 = -.02x$$

$$\frac{p - 200}{-.02} = x$$

$$\frac{200 - p}{.02} = x$$

When $\bar{p} = 150$, the corresponding value of $\bar{x}$ is

x-intercept is

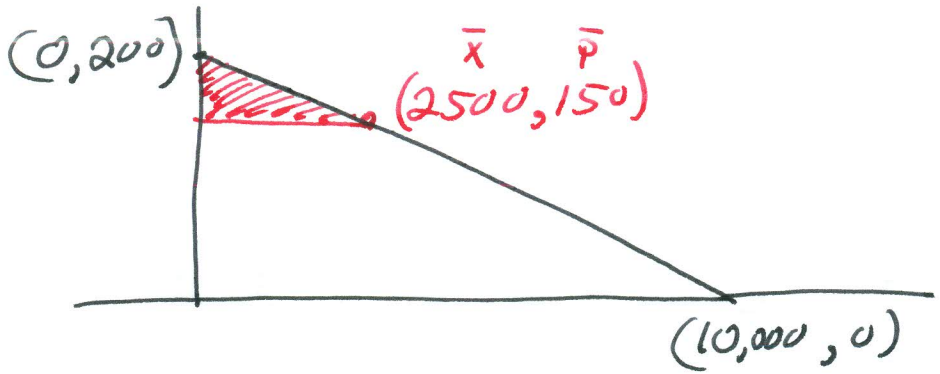
Set $p = 0$, solve for x

$$0 = 200 - .02x$$

$$.02x = 200$$

$$x = \frac{200}{.02} = 10{,}000$$

$$\bar{x} = \frac{200 - 150}{.02} = \frac{50}{.02} = 2500$$



CS = area of triangle 

$$= \frac{1}{2} b \cdot h = \frac{1}{2}(2500)(50) = (2500)(25) = \$62{,}500$$

Consumer's surplus