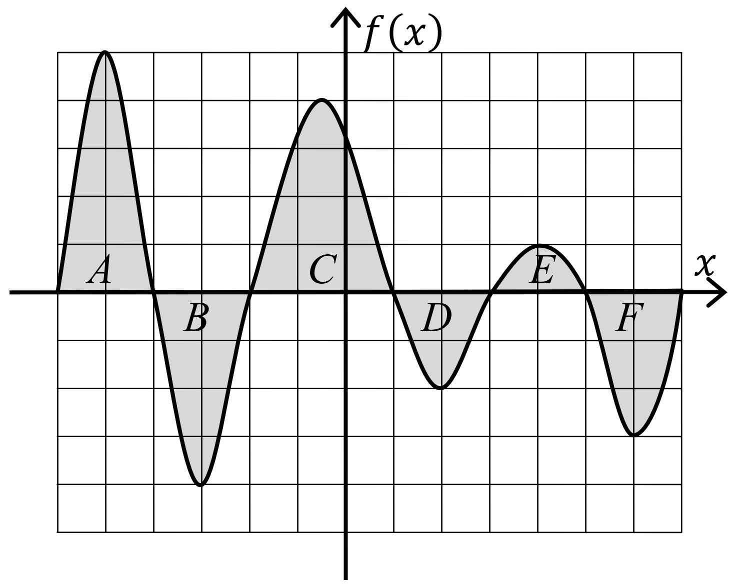
2011 - 2012 Fall Semester MATH 1350 (Barsamian) Quiz 9 Version 1 Solutions

(Version 1 has a 5 in problem [2].)

[1] The graph of $f(x)$ is shown at right. The areas of the six shaded regions are:
The area of region A is 6.
The area of region B is 5.
The area of region C is 7.
The area of region D is 3.
The area of region E is 2.
The area of region F is 4.



Find the value of the definite integral:

$$\int_{x=-6}^{x=3} f(x)dx =$$

Solution: $\int_{x=-6}^{x=3} f(x)dx$ = signed area from x = -6 to x = 3

= 6 – 5 + 7 - 3

= 5

[2] Find $F(x) = \int 5x(x^2+29)^3 dx$. Show all details clearly.

Solution #1: The method that I have been using in class:

Identify stuff:	integrand $f(x) = 5x(x^2+29)^3$ $inner(x) = x^2 + 29$ $outer'(\quad) = (\quad)^3$
Find the outer function:	$outer(\quad)$ will be the antiderivative of $(\quad)^3$. Using the power rule for antiderivatives, we get $outer(\quad) = \frac{(\quad)^{3+1}}{3+1} = \frac{(\quad)^4}{4}$.
First Try:	Try $F(x) = outer\big(inner(x)\big) = \frac{(x^2+29)^4}{4}$
Check by taking derivative:	$F'(x) = \frac{d}{dx}\frac{(x^2+29)^4}{4} = \left(\frac{1}{4}\right)\frac{d}{dx}(x^2+29)^4 = \left(\frac{1}{4}\right)(4(x^2+29)^3 \cdot 2x)$ $= (x^2+29)^3 \cdot 2x = \left(\frac{2}{5}\right)(5x(x^2+29)^3) = \left(\frac{2}{5}\right)f(x)$ Conclude that we are off by a factor of $\left(\frac{2}{5}\right)$.
Second Try:	Try $F(x) = \left(\frac{5}{2}\right)\frac{(x^2+29)^4}{4}$.
Check by taking derivative:	$F'(x) = \frac{d}{dx}\left(\frac{5}{2}\right)\frac{(x^2+29)^4}{4} = \left(\frac{5}{2}\right)\frac{d}{dx}\frac{(x^2+29)^4}{4} = \left(\frac{5}{2}\right)\left(\frac{2}{5}\right)f(x) = f(x)$. Since $F'(x) = f(x)$, we conclude that our second try was the correct $F(x)$.
Conclusion:	The General Antiderivative is $F(x) = \frac{5(x^2+29)^4}{8} + C$.

Solution #2: The Book's method for finding $F(x) = \int 5x(x^2+29)^3 dx$.

Identify the integrand $f(x) = 5x(x^2+29)^3$.

Let $u = x^2 + 29$. With this, we have a new version of $(x^2+29)^3$. We can replace it with u^3.

We still need to find a new version of $5xdx$.

Take $\frac{d}{dx}$ of both sides of the equation $u = x^2 + 29$. The result is the new equation

$$\frac{du}{dx} = \frac{d}{dx}(x^2+29)$$
$$\frac{du}{dx} = 2x$$

Multiply both sides of this equation by dx. The result is the new equation

$$du = 2xdx$$

This is not quite what we need. So multiply both sides of this equation by $\left(\frac{5}{2}\right)$. The result is the new equation

$$\left(\frac{5}{2}\right)du = \left(\frac{5}{2}\right)2xdx$$
$$\left(\frac{5}{2}\right)du = 5xdx$$

So now, we have a new version of $(x^2+29)^3$ and we have a new version of $5xdx$. Substituting in the new versions, the antiderivative becomes

$$\int u^3\left(\frac{5}{2}\right)du$$

We find this antiderivative using the basic antiderivative rules:

$$\int u^3\left(\frac{5}{2}\right)du = \left(\frac{5}{2}\right)\int u^3 du = \left(\frac{5}{2}\right)\frac{u^{3+1}}{3+1} + C = \left(\frac{5}{2}\right)\frac{u^4}{4} + C = \frac{5u^4}{8} + C$$

Substituting back, we get

$$F(x) = \frac{5(x^2+29)^4}{8} + C$$

Check by taking the derivative:

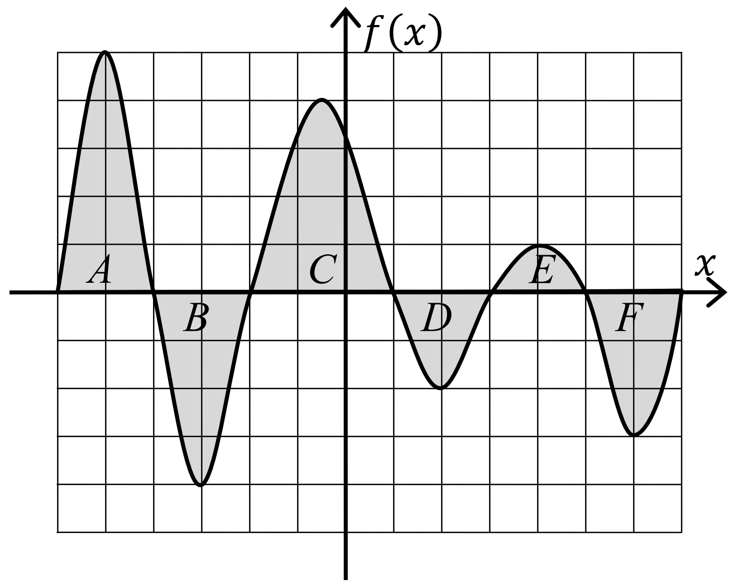
$$F'(x) = \frac{d}{dx}\left(\frac{5(x^2+29)^4}{8} + C\right) = \frac{5}{8}\frac{d}{dx}(x^2+29)^4 + 0 = \frac{5}{8}\left((4)(x^2+29)^3 \cdot 2x\right) = 5x(x^2+29)^3 = f(x)$$

Since $F'(x) = f(x)$, we conclude that $F(x) = \frac{5(x^2+29)^4}{8} + C$ is correct.

2011 - 2012 Fall Semester MATH 1350 (Barsamian) Quiz 9 Version 2 Solutions

(Version 2 has a 7 in problem [2].)

[1] The graph of $f(x)$ is shown at right. The areas of the six shaded regions are:
The area of region A is 6.
The area of region B is 5.
The area of region C is 7.
The area of region D is 3.
The area of region E is 2.
The area of region F is 4.



Find the value of the definite integral:

$$\int_{x=-2}^{x=7} f(x)dx =$$

Solution: $\int_{x=-2}^{x=7} f(x)dx$ = signed area from x = -2 to x = 7

= 7 – 3 + 2 - 4

= 2

[2] Find $F(x) = \int 7x(x^2+23)^3 dx$. Show all details clearly.

Solution #1: The method that I have been using in class:

Identify stuff:	integrand $f(x) = 7x(x^2+29)^3$ $inner(x) = x^2 + 29$ $outer'(\quad) = (\quad)^3$
Find the outer function:	$outer(\quad)$ will be the antiderivative of $(\quad)^3$. Using the power rule for antiderivatives, we get $outer(\quad) = \frac{(\quad)^{3+1}}{3+1} = \frac{(\quad)^4}{4}$.
First Try:	Try $F(x) = outer\big(inner(x)\big) = \frac{(x^2+29)^4}{4}$
Check by taking derivative:	$F'(x) = \frac{d}{dx}\frac{(x^2+29)^4}{4} = \left(\frac{1}{4}\right)\frac{d}{dx}(x^2+29)^4 = \left(\frac{1}{4}\right)(4(x^2+29)^3 \cdot 2x)$ $= (x^2+29)^3 \cdot 2x = \left(\frac{2}{7}\right)(7x(x^2+29)^3) = \left(\frac{2}{7}\right)f(x)$ Conclude that we are off by a factor of $\left(\frac{2}{7}\right)$.
Second Try:	Try $F(x) = \left(\frac{7}{2}\right)\frac{(x^2+29)^4}{4}$.
Check by taking derivative:	$F'(x) = \frac{d}{dx}\left(\frac{7}{2}\right)\frac{(x^2+29)^4}{4} = \left(\frac{7}{2}\right)\frac{d}{dx}\frac{(x^2+29)^4}{4} = \left(\frac{7}{2}\right)\left(\frac{2}{7}\right)f(x) = f(x)$. Since $F'(x) = f(x)$, we conclude that our second try was the correct $F(x)$.
Conclusion:	The General Antiderivative is $F(x) = \frac{7(x^2+29)^4}{8} + C$.

Solution #2: The Book's method for finding $F(x) = \int 7x(x^2 + 29)^3 dx$.

Identify the integrand $f(x) = 7x(x^2 + 29)^3$.

Let $u = x^2 + 29$. With this, we have a new version of $(x^2 + 29)^3$. We can replace it with u^3.

We still need to find a new version of $7xdx$.

Take $\frac{d}{dx}$ of both sides of the equation $u = x^2 + 29$. The result is the new equation

$$\frac{du}{dx} = \frac{d}{dx}(x^2 + 29)$$
$$\frac{du}{dx} = 2x$$

Multiply both sides of this equation by dx. The result is the new equation

$$du = 2xdx$$

This is not quite what we need. So multiply both sides of this equation by $\left(\frac{7}{2}\right)$. The result is the new equation

$$\left(\frac{7}{2}\right) du = \left(\frac{7}{2}\right) 2xdx$$
$$\left(\frac{7}{2}\right) du = 5xdx$$

So now, we have a new version of $(x^2 + 29)^3$ and we have a new version of $7xdx$. Substituting in the new versions, the antiderivative becomes

$$\int u^3 \left(\frac{7}{2}\right) du$$

We find this antiderivative using the basic antiderivative rules:

$$\int u^3 \left(\frac{7}{2}\right) du = \left(\frac{7}{2}\right) \int u^3 du = \left(\frac{7}{2}\right) \frac{u^{3+1}}{3+1} + C = \left(\frac{7}{2}\right) \frac{u^4}{4} + C = \frac{7u^4}{8} + C$$

Substituting back, we get

$$F(x) = \frac{7(x^2 + 29)^4}{8} + C$$

Check by taking the derivative:

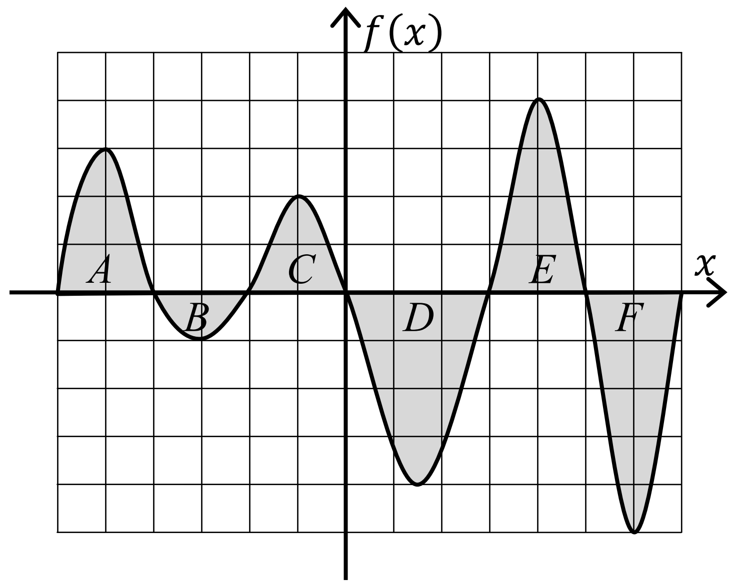
$$F'(x) = \frac{d}{dx}\left(\frac{7(x^2 + 29)^4}{8} + C\right) = \frac{7}{8}\frac{d}{dx}(x^2 + 29)^4 + 0 = \frac{7}{8}\big((4)(x^2 + 29)^3 \cdot 2x\big) = 7x(x^2 + 29)^3 = f(x)$$

Since $F'(x) = f(x)$, we conclude that $F(x) = \frac{7(x^2+29)^4}{8} + C$ is correct.

2011 - 2012 Fall Semester MATH 1350 (Barsamian) Quiz 9 Version 3 Solutions

(Version 3 has an 11 in problem [2].)

[1] The graph of $f(x)$ is shown at right. The areas of the six shaded regions are:
The area of region A is 4.
The area of region B is 2.
The area of region C is 3.
The area of region D is 7.
The area of region E is 5.
The area of region F is 6.



Find the value of the definite integral:

$$\int_{x=-6}^{x=3} f(x)dx =$$

Solution: $\int_{x=-6}^{x=3} f(x)dx$ = signed area from x = -6 to x = 3

= 4 – 2 + 3 - 7

= -2

[2] Find $F(x) = \int 11x(x^2+19)^3 dx$. Show all details clearly.

Solution #1: The method that I have been using in class:

Identify stuff:	integrand $f(x) = 11x(x^2+29)^3$ $inner(x) = x^2+29$ $outer'(\quad) = (\quad)^3$
Find the outer function:	$outer(\quad)$ will be the antiderivative of $(\quad)^3$. Using the power rule for antiderivatives, we get $outer(\quad) = \frac{(\quad)^{3+1}}{3+1} = \frac{(\quad)^4}{4}$.
First Try:	Try $F(x) = outer\big(inner(x)\big) = \frac{(x^2+29)^4}{4}$
Check by taking derivative:	$F'(x) = \frac{d}{dx}\frac{(x^2+29)^4}{4} = \left(\frac{1}{4}\right)\frac{d}{dx}(x^2+29)^4 = \left(\frac{1}{4}\right)(4(x^2+29)^3 \cdot 2x)$ $= (x^2+29)^3 \cdot 2x = \left(\frac{2}{11}\right)(11x(x^2+29)^3) = \left(\frac{2}{11}\right)f(x)$ Conclude that we are off by a factor of $\left(\frac{2}{11}\right)$.
Second Try:	Try $F(x) = \left(\frac{11}{2}\right)\frac{(x^2+29)^4}{4}$.
Check by taking derivative:	$F'(x) = \frac{d}{dx}\left(\frac{11}{2}\right)\frac{(x^2+29)^4}{4} = \left(\frac{11}{2}\right)\frac{d}{dx}\frac{(x^2+29)^4}{4} = \left(\frac{11}{2}\right)\left(\frac{2}{11}\right)f(x) = f(x)$. Since $F'(x) = f(x)$, we conclude that our second try was the correct $F(x)$.
Conclusion:	The General Antiderivative is $F(x) = \frac{11(x^2+29)^4}{8} + C$.

Solution #2: The Book's method for finding $F(x) = \int 11x(x^2 + 29)^3 dx$.

Identify the integrand $f(x) = 11x(x^2 + 29)^3$.

Let $u = x^2 + 29$. With this, we have a new version of $(x^2 + 29)^3$. We can replace it with u^3.

We still need to find a new version of $11xdx$.

Take $\frac{d}{dx}$ of both sides of the equation $u = x^2 + 29$. The result is the new equation

$$\frac{du}{dx} = \frac{d}{dx}(x^2 + 29)$$
$$\frac{du}{dx} = 2x$$

Multiply both sides of this equation by dx. The result is the new equation

$$du = 2xdx$$

This is not quite what we need. So multiply both sides of this equation by $\left(\frac{11}{2}\right)$. The result is the new equation

$$\left(\frac{11}{2}\right) du = \left(\frac{11}{2}\right) 2xdx$$
$$\left(\frac{11}{2}\right) du = 11xdx$$

So now, we have a new version of $(x^2 + 29)^3$ and we have a new version of $11xdx$. Substituting in the new versions, the antiderivative becomes

$$\int u^3 \left(\frac{11}{2}\right) du$$

We find this antiderivative using the basic antiderivative rules:

$$\int u^3 \left(\frac{11}{2}\right) du = \left(\frac{11}{2}\right) \int u^3 du = \left(\frac{11}{2}\right) \frac{u^{3+1}}{3+1} + C = \left(\frac{11}{2}\right) \frac{u^4}{4} + C = \frac{11u^4}{8} + C$$

Substituting back, we get

$$F(x) = \frac{11(x^2 + 29)^4}{8} + C$$

Check by taking the derivative:

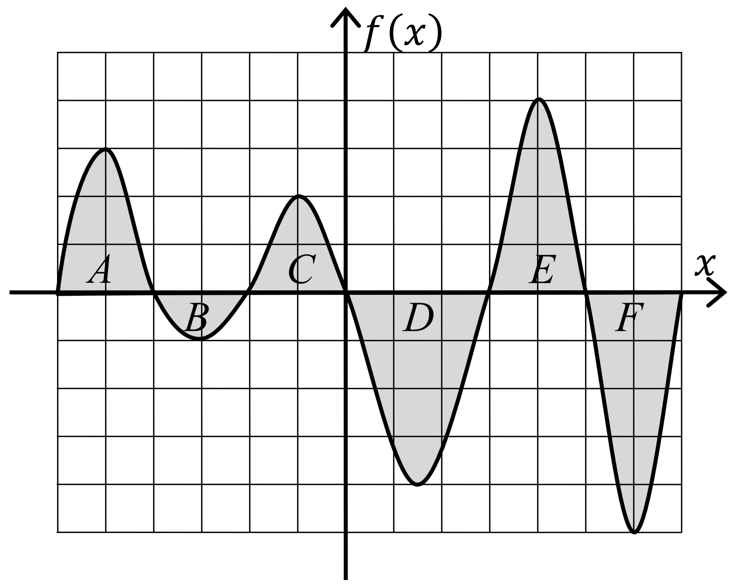
$$F'(x) = \frac{d}{dx}\left(\frac{11(x^2 + 29)^4}{8} + C\right) = \frac{11}{8}\frac{d}{dx}(x^2 + 29)^4 + 0 = \frac{11}{8}\left((4)(x^2 + 29)^3 \cdot 2x\right) = 11x(x^2 + 29)^3$$
$$= f(x)$$

Since $F'(x) = f(x)$, we conclude that $F(x) = \frac{11(x^2+29)^4}{8} + C$ is correct.

2011 - 2012 Fall Semester MATH 1350 (Barsamian) Quiz 9 Version 4 Solutions

(Version 4 has a 13 in problem [2].)

[1] The graph of $f(x)$ is shown at right. The areas of the six shaded regions are:
The area of region A is 4.
The area of region B is 2.
The area of region C is 3.
The area of region D is 7.
The area of region E is 5.
The area of region F is 6.



Find the value of the definite integral:

$$\int_{x=-2}^{x=7} f(x)dx =$$

Solution: $\int_{x=-2}^{x=7} f(x)dx$ = signed area from x = -2 to x = 7

= 3 – 7 + 5 - 6

= -5

[2] Find $F(x) = \int 13x(x^2 + 17)^3 dx$. Show all details clearly.

Solution #1: The method that I have been using in class:

Identify stuff:	integrand $f(x) = 13x(x^2 + 29)^3$ $inner(x) = x^2 + 29$ $outer'(\quad) = (\quad)^3$
Find the outer function:	$outer(\quad)$ will be the antiderivative of $(\quad)^3$. Using the power rule for antiderivatives, we get $outer(\quad) = \frac{(\quad)^{3+1}}{3+1} = \frac{(\quad)^4}{4}$.
First Try:	Try $F(x) = outer\big(inner(x)\big) = \frac{(x^2+29)^4}{4}$
Check by taking derivative:	$F'(x) = \frac{d}{dx}\frac{(x^2+29)^4}{4} = \left(\frac{1}{4}\right)\frac{d}{dx}(x^2+29)^4 = \left(\frac{1}{4}\right)(4(x^2+29)^3 \cdot 2x)$ $= (x^2+29)^3 \cdot 2x = \left(\frac{2}{13}\right)(13x(x^2+29)^3) = \left(\frac{2}{13}\right)f(x)$ Conclude that we are off by a factor of $\left(\frac{2}{13}\right)$.
Second Try:	Try $F(x) = \left(\frac{13}{2}\right)\frac{(x^2+29)^4}{4}$.
Check by taking derivative:	$F'(x) = \frac{d}{dx}\left(\frac{13}{2}\right)\frac{(x^2+29)^4}{4} = \left(\frac{13}{2}\right)\frac{d}{dx}\frac{(x^2+29)^4}{4} = \left(\frac{13}{2}\right)\left(\frac{2}{13}\right)f(x) = f(x)$. Since $F'(x) = f(x)$, we conclude that our second try was the correct $F(x)$.
Conclusion:	The General Antiderivative is $F(x) = \frac{13(x^2+29)^4}{8} + C$.

Solution #2: The Book's method for finding $F(x) = \int 13x(x^2+29)^3 dx$.

Identify the integrand $f(x) = 13x(x^2+29)^3$.

Let $u = x^2 + 29$. With this, we have a new version of $(x^2+29)^3$. We can replace it with u^3.

We still need to find a new version of $13xdx$.

Take $\frac{d}{dx}$ of both sides of the equation $u = x^2 + 29$. The result is the new equation

$$\frac{du}{dx} = \frac{d}{dx}(x^2+29)$$
$$\frac{du}{dx} = 2x$$

Multiply both sides of this equation by dx. The result is the new equation

$$du = 2xdx$$

This is not quite what we need. So multiply both sides of this equation by $\left(\frac{13}{2}\right)$. The result is the new equation

$$\left(\frac{13}{2}\right) du = \left(\frac{13}{2}\right) 2xdx$$
$$\left(\frac{13}{2}\right) du = 13xdx$$

So now, we have a new version of $(x^2+29)^3$ and we have a new version of $13xdx$. Substituting in the new versions, the antiderivative becomes

$$\int u^3 \left(\frac{13}{2}\right) du$$

We find this antiderivative using the basic antiderivative rules:

$$\int u^3 \left(\frac{13}{2}\right) du = \left(\frac{13}{2}\right) \int u^3 du = \left(\frac{13}{2}\right) \frac{u^{3+1}}{3+1} + C = \left(\frac{13}{2}\right) \frac{u^4}{4} + C = \frac{13u^4}{8} + C$$

Substituting back, we get

$$F(x) = \frac{13(x^2+29)^4}{8} + C$$

Check by taking the derivative:

$$F'(x) = \frac{d}{dx}\left(\frac{13(x^2+29)^4}{8} + C\right) = \frac{13}{8}\frac{d}{dx}(x^2+29)^4 + 0 = \frac{13}{8}\big((4)(x^2+29)^3 \cdot 2x\big) = 13x(x^2+29)^3$$
$$= f(x)$$

Since $F'(x) = f(x)$, we conclude that $F(x) = \frac{13(x^2+29)^4}{8} + C$ is correct.