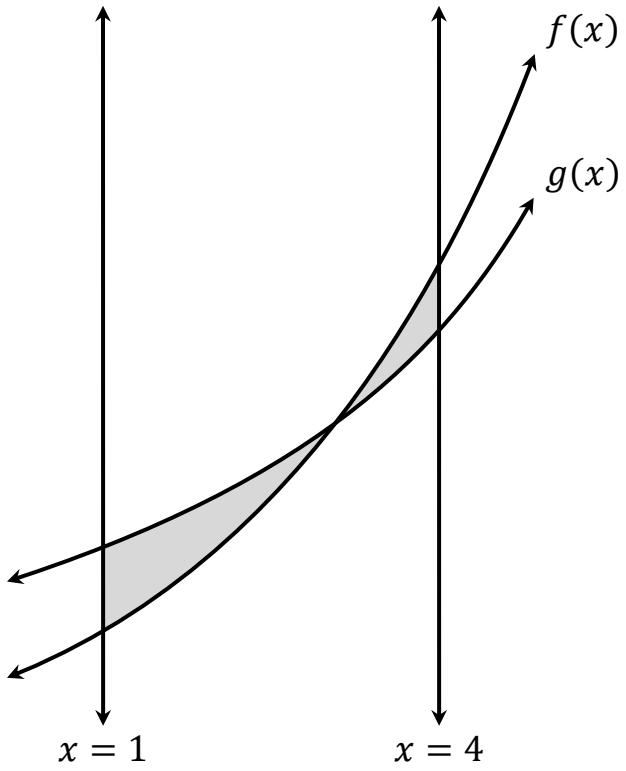
2011 - 2012 Fall Semester MATH 1350 (Barsamian) Quiz 10 Solutions

The goal is to find the unsigned area (USA) of the region bordered by the two curves and two lines listed below:

- the curve $f(x) = x^2 + 8x - 7$
- the curve $g(x) = x^2 + 6x - 1$
- the line $x = 1$
- the line $x = 4$

The region is shaded in the drawing at right.



(A) Use algebra to find the *x*-coordinate of the point where the two curves intersect.
(Hint: set $f(x) = g(x)$ and solve for x.)

Solution:

$$\begin{aligned} f(x) &= g(x) \\ x^2 + 8x - 7 &= x^2 + 6x - 1 \\ 8x - 6x &= -1 + 7 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

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(B) Set up a sum of definite integrals to compute the unsigned area. It should look like this.

$$USA = \int_{x=a}^{x=b} (some\ \ integrand\ here)dx + \int_{x=b}^{x=c} (another\ \ integrand\ here)dx$$

(You will have to figure out the integrands and the limits of integration a, b, c.)

(C) Use calculus to find the value of the definite integrals and find their sum, USA.
(Be sure to simplify the integrands before integrating!)

Solutions to (B) and (C):

$$\begin{aligned} USA &= \int_{x=1}^{x=3} \big(g(x) - f(x)\big)dx + \int_{x=3}^{x=4} \big(f(x) - g(x)\big)dx \\ &= \int_{x=1}^{x=3} \big((x^2 + 6x - 1) - (x^2 + 8x - 7)\big)dx + \int_{x=3}^{x=4} \big((x^2 + 8x - 7) - (x^2 + 6x - 1)\big)dx \\ &= \int_{x=1}^{x=3} (-2x + 6)dx + \int_{x=3}^{x=4} (2x - 6)dx \\ &= (-x^2 + 6x)\Big|_{x=1}^{x=3} + (x^2 - 6x)\Big|_{x=3}^{x=4} \\ &= \big(\big(-(3)^2 + 6(3)\big) - \big(-(1)^2 + 6(1)\big)\big) + \big((4)^2 - 6(4)\big) - \big((3)^2 - 6(3)\big) \\ &= \big((-9 + 18) - (-1 + 6)\big) + \big((16 - 24) - (9 - 18)\big) \\ &= (9 - 5) + \big(-8 - (-9)\big) \\ &= 4 + 1 \\ &= 5 \end{aligned}$$