

Reference 5: Facts About Limits from Section 3-1

Suppose that $\lim_{x \rightarrow c} f(x) = L$ and $\lim_{x \rightarrow c} g(x) = M$ exist, where L and M are real numbers. Then

Theorem 2.1:	$\lim_{x \rightarrow c} k = k$
Theorem 2.2:	$\lim_{x \rightarrow c} x = c$
Theorem 2.3:	$\lim_{x \rightarrow c} [f(x) + g(x)] = \lim_{x \rightarrow c} f(x) + \lim_{x \rightarrow c} g(x) = L + M$
Theorem 2.4:	$\lim_{x \rightarrow c} [f(x) - g(x)] = \lim_{x \rightarrow c} f(x) - \lim_{x \rightarrow c} g(x) = L - M$
Theorem 2.5:	If k is a constant, then $\lim_{x \rightarrow c} kf(x) = k \lim_{x \rightarrow c} f(x) = kL$.
Theorem 2.6:	$\lim_{x \rightarrow c} (f(x) \cdot g(x)) = (\lim_{x \rightarrow c} f(x)) \cdot (\lim_{x \rightarrow c} g(x)) = L \cdot M$
Theorem 2.7:	If $\lim_{x \rightarrow c} g(x) = M \neq 0$, then $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)} = \frac{L}{M}$
Theorem 2.8:	If n is a positive odd integer, or if n is a positive even integer and $\lim_{x \rightarrow c} f(x) = L > 0$, then $\lim_{x \rightarrow c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow c} f(x)}$.
Theorem 3:	If f is a polynomial, then $\lim_{x \rightarrow c} f(x) = f(c)$. If r is a rational function and c is in the domain of r , then $\lim_{x \rightarrow c} r(x) = r(c)$.
Definition:	If $\lim_{x \rightarrow c} f(x) = L = 0$ and $\lim_{x \rightarrow c} g(x) = M = 0$, then $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$ is said to have the <i>indeterminate form</i> $\frac{0}{0}$. In this case, Theorem 2.7 cannot be used to determine the limit.
Theorem 4:	If $\lim_{x \rightarrow c} f(x) = L \neq 0$ and $\lim_{x \rightarrow c} g(x) = M = 0$, then $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$ does not exist according to the Definition of the Limit found in Section 3-1. (Remark: In Section 3-2, the Definition of Limit gets expanded, with the result that in some cases, we will say that the limit is infinity or negative infinity and will write the symbol $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \infty$ or $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = -\infty$. But this will not always be the case. That is, sometimes the limit will not exist even with the expanded Definition of Limit.)
One x Rule (not in book)	If f and g are functions whose y-values differ at only one x-value, c (that is, the y-values are the same for $x \neq c$ but differ at $x = c$), then $\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} g(x)$.