

Class Drill 4: Representations of Slopes

In Section 3-4 of the textbook, you learned about average rate of change and instantaneous rate of change.

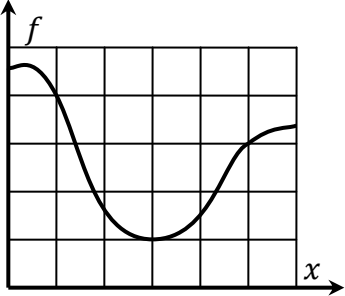
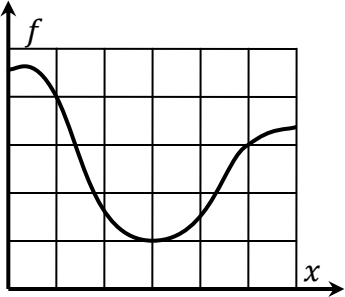
Definition of Average Rate of Change

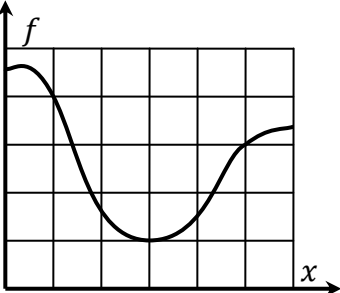
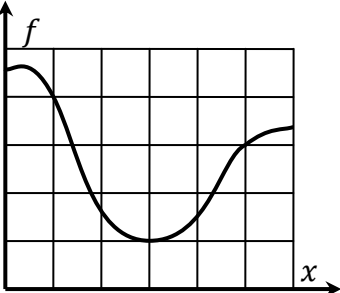
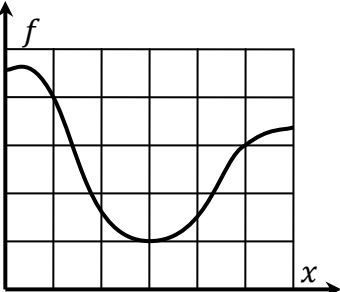
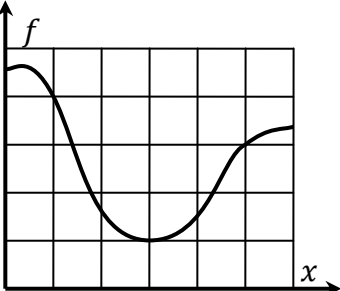
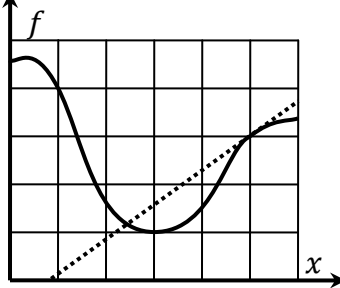
- **words:** the average rate of change of f as the input changes from a to b
- **usage:** f is a function that is continuous on the interval $[a, b]$.
- **meaning:** the number $m = \frac{f(b)-f(a)}{b-a}$
- **graphical interpretation:** The number m is the slope of the secant line that touches the graph of f at the points $(a, f(a))$ and $(b, f(b))$.
- **remark:** The average rate of change m is a number.

Definition of Instantaneous Rate of Change

- **words:** the instantaneous rate of change of f at a
- **alternate words:** the derivative of f at a
- **symbol:** $f'(a)$
- **meaning:** the number $m = \lim_{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$
- **graphical interpretation:** The number m is the slope of the line tangent to the graph of f at the point $(x, y) = (a, f(a))$.
- **remark:** The instantaneous rate of change $f'(a)$ is a number.

Each expression in the left column represents a number m that can be interpreted as the slope of a line on the graph of f . In each example, draw the line on the graph of f , or write the missing expression based on the line shown in the graph, and then give the value of the number m represented by the expression.

| <u>Example</u> | <u>Expression representing m</u> | <u>Line whose slope is m</u> | <u>Value of m</u> |
|----------------|--|--|--------------------------------|
| (1) | the average rate of change of f as the input changes from 1 to 5 |  | $m =$ |
| (2) | the derivative of f at $x = 1$ |  | $m =$ |

| Example | Expression representing m | Line whose slope is m | Value of m |
|---------|--|--|--------------|
| (3) | the instantaneous rate of change of f at $x = 4$ |  | $m =$ |
| (4) | $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$ |  | $m =$ |
| (5) | $\frac{f(4) - f(2)}{4 - 2}$ |  | $m =$ |
| (6) | $f'(2)$ |  | $m =$ |
| (7) | |  | $m =$ |