

Class Drill 17: Finding the Signed Area Under a Graph By Two Different Methods

Part I: Drawing

Let $f(x) = 3x - 6$.

(a) Draw the graph of f for $0 \leq x \leq 6$. Make your graph large and neat. Find the coordinates of the x -intercept on the graph and label that.

(b) On your graph, shade the region between the graph of f and the x -axis from $x = 1$ to $x = 5$. The shaded region should be made up of two triangles.

Method #1 Finding Areas Using Geometry

(c) Using the geometric formula for the area of a triangle, find the area of each of the two triangles.

(d) Using the known areas of the two triangles, find the area of the shaded region. (The **unsigned area**.) It should be the sum of the positive numbers that are the areas of the two triangles.

(e) Using the known areas of the two triangles, find the **signed area** of the shaded region. That is, find the value of

$$A = \int_{x=1}^{x=5} f(x)dx = \int_{x=1}^{x=5} 3x - 6dx$$

It should be the difference of the positive numbers that are the areas of the two triangles.

Method #2 Finding The Signed Area Using Calculus

(f) Use the antiderivative formulas to find an antiderivative $F(x)$ for $f(x)$. That is, use the antiderivative formulas to find

$$F(x) = \int f(x)dx = \int 3x - 6dx$$

(g) Check: Does $f'(x) = f(x)$? If not, then go back to step (f) and check your work.

(h) Using the function $F(x)$ that you found, compute $F(5) - F(1)$.

Comparing The Numbers Obtained By The Two Methods

(i) Does your answer to question (e) match your answer to question (h)?

That is, does $A = F(5) - F(1)$?

That is, is the following equation true?

$$\int_{x=1}^{x=5} 3x - 6dx = F(5) - F(1)$$