

# 2012-2013 Spring Semester Math 1350

Day 1, Monday, January 14, 2013

## Chapter 3

### Section 3-1 Limits

#### The Definition of Limit

- Symbol:  $\lim_{x \rightarrow c} f(x) = L$

• spoken: "The limit, as  $x$  approaches  $c$ , of  $f(x)$  is  $L$ ."

• less-abbreviated symbols:  $f(x) \rightarrow L$  as  $x \rightarrow c$

• spoken: " $f(x)$  approaches  $L$  as  $x$  approaches  $c$ ."

• usage:  $x$  is a variable,  $f$  is a function

$c$  is a real number constant,  $L$  is a real number constant

• meaning: As  $x$  gets closer + closer to  $c$ , but not equal to  $c$ ,  
the value of  $f(x)$  gets closer + closer to  $L$ . (And may equal  $L$ .)

• graphical interpretation: The graph of  $f(x)$  appears to be  
heading for the location  $(x, y) = (c, L)$ .

## Today: Graphical Approach to Limits

Class Drill 1 Limits on page 8 of the course packet.

Start with the row where  $x=1$

$f(1)$  DNE (does not exist) because there is  $\sim$  point on the graph at  $x=1$ . (There is a hole)

What about  $\lim_{x \rightarrow 1} f(x) = ??$

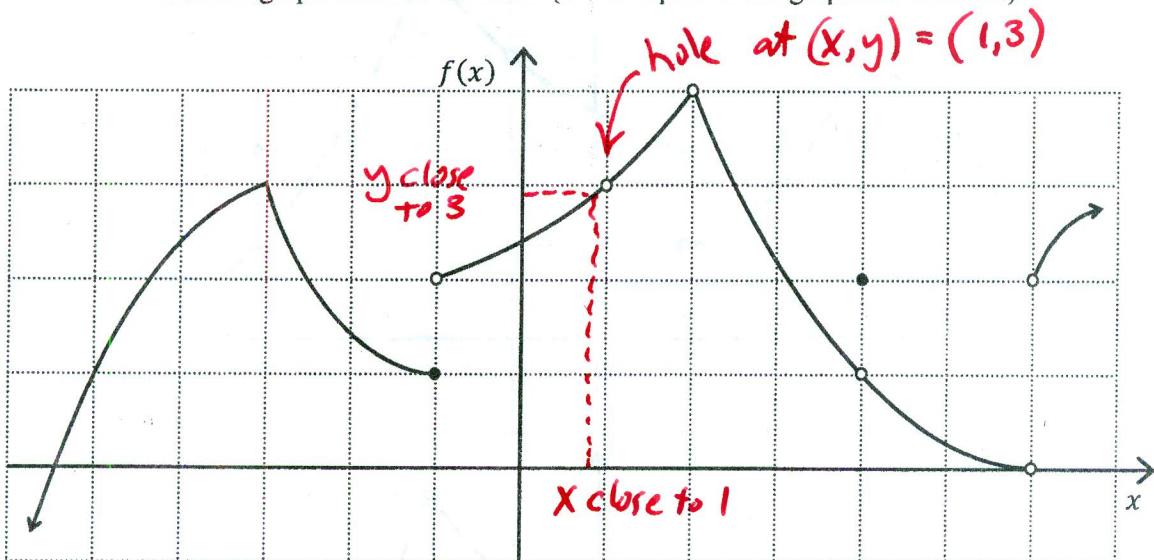
When  $x$  is close to 1 but not equal to 1,  $y$  gets close to 3.

In symbols, this gets abbreviated  $\lim_{x \rightarrow 1} f(x) = 3$

In other words, the graph appears to be heading for the location  $(x, y) = (1, 3)$ .

### Class Drill 1: Limits

Use the graph to fill in the table. (Extra copies of the graph are on back.)



x-value	limit from left	limit from right	limit	y-value
-5	$\lim_{x \rightarrow -5^-} f(x) =$	$\lim_{x \rightarrow -5^+} f(x) =$	$\lim_{x \rightarrow -5} f(x) =$	$f(-5) =$
-3	$\lim_{x \rightarrow -3^-} f(x) =$	$\lim_{x \rightarrow -3^+} f(x) =$	$\lim_{x \rightarrow -3} f(x) =$	$f(-3) =$
-1	$\lim_{x \rightarrow -1^-} f(x) =$	$\lim_{x \rightarrow -1^+} f(x) =$	$\lim_{x \rightarrow -1} f(x) =$	$f(-1) =$
1	$\lim_{x \rightarrow 1^-} f(x) =$	$\lim_{x \rightarrow 1^+} f(x) =$	$\lim_{x \rightarrow 1} f(x) =$ 3	$f(1) =$ DNE
2	$\lim_{x \rightarrow 2^-} f(x) =$	$\lim_{x \rightarrow 2^+} f(x) =$	$\lim_{x \rightarrow 2} f(x) =$	$f(2) =$
4	$\lim_{x \rightarrow 4^-} f(x) =$	$\lim_{x \rightarrow 4^+} f(x) =$	$\lim_{x \rightarrow 4} f(x) =$	$f(4) =$ 2
6	$\lim_{x \rightarrow 6^-} f(x) =$	$\lim_{x \rightarrow 6^+} f(x) =$	$\lim_{x \rightarrow 6} f(x) =$	$f(6) =$

Now do the row of the table with  $x=4$

$f(4)=2$  because there is a dot at  $(x,y)=(4,2)$ .

But when  $x$  is close to 4 but not equal to 4,  
the  $y$ -values get closer + closer to 1.

That is the graph appears to be heading for the  
location  $(x,y)=(4,1)$ . (Hole at  $(x,y)=(4,1)$ )

In symbols,  $\lim_{x \rightarrow 4} f(x) = 1$

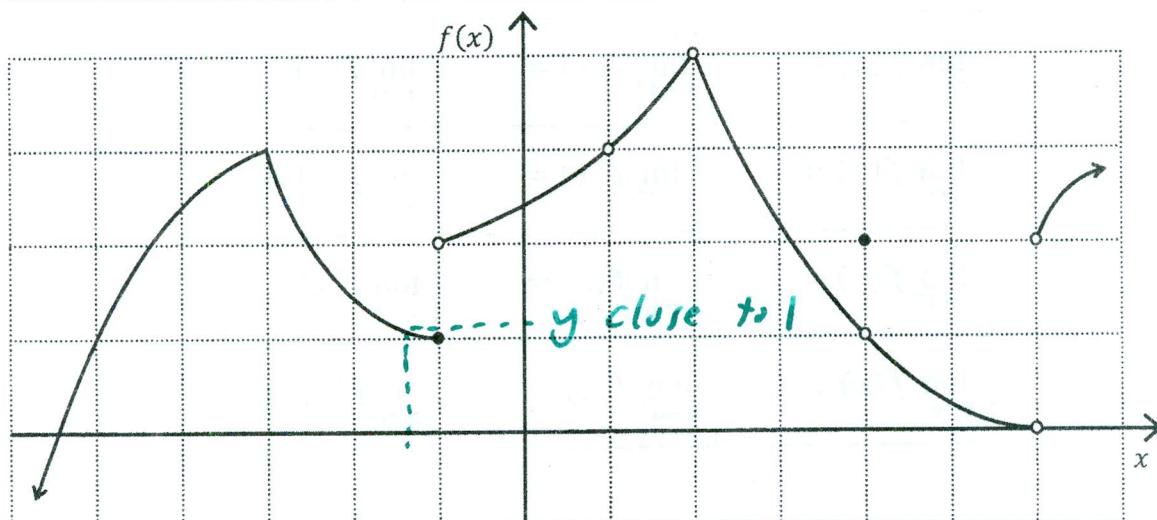
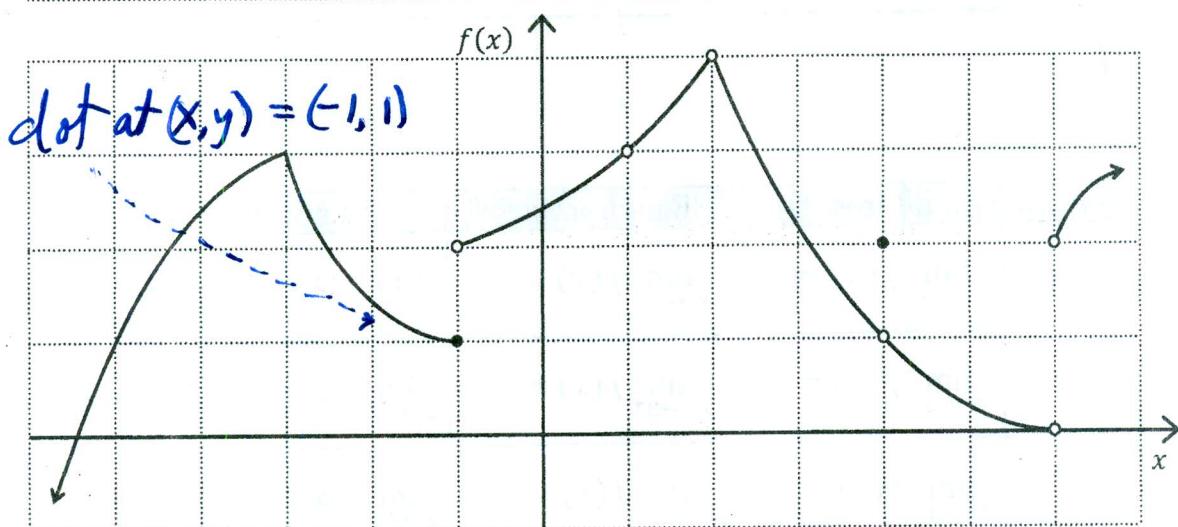
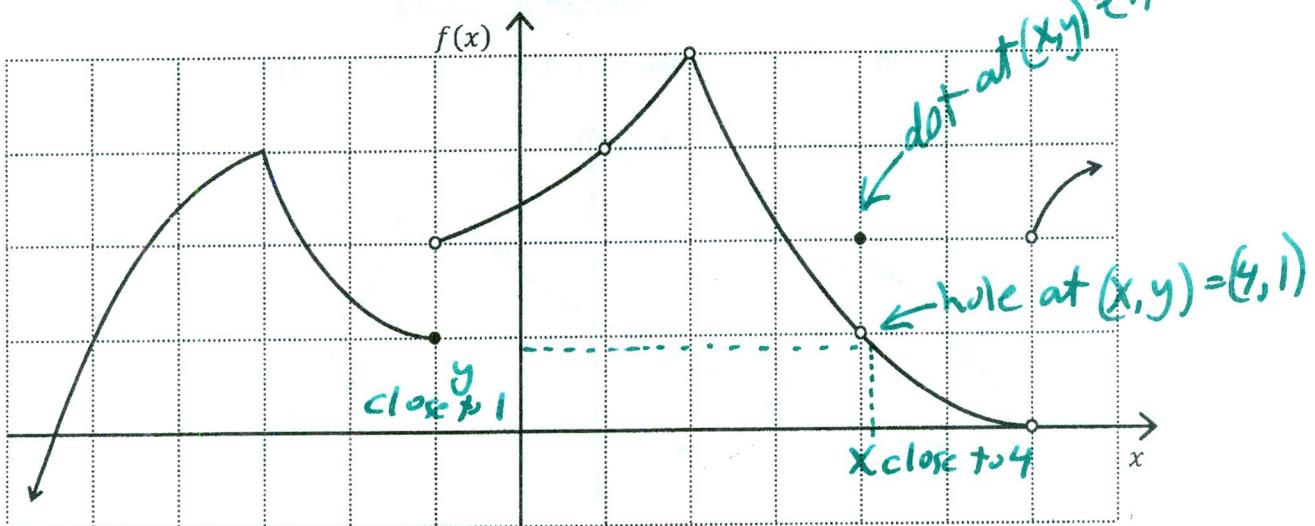
Now do the row with  $x=-1$ ,

$f(-1)=1$  because graph has dot at  $(x,y)=(-1,1)$

What about  $\lim_{x \rightarrow -1}$ ?

When  $x$  is close to -1, but to left of -1,  $y$  values are close to  $y=1$ .

This is ~~abbreviate~~ related to "one-sided limits"



$X$  value close to  $-1$   
but to left of  $-1$

$$\lim_{x \rightarrow -1^-} f(x) = 1$$

That is, the "limit from the left" is the number 1.

Symbol:  $\lim_{x \rightarrow -1^-} f(x) = 1$

↑ minus sign denotes limit  
from the left.