Day 2 is Tuesday, January 15, 2013

Resume Class Drill 1 Limits

Working on row where $x = -1$

We have already found $f(-1) = 1$ because dot at $(-1, 1)$

We have discussed the fact that

when x is close to -1 but $\boxed{\text{to left}}$ of -1, y is close to 1.
This brings up the concept of the "one-sided-limit"

Symbol $\lim_{x \to -1^-} f(x) = 1$

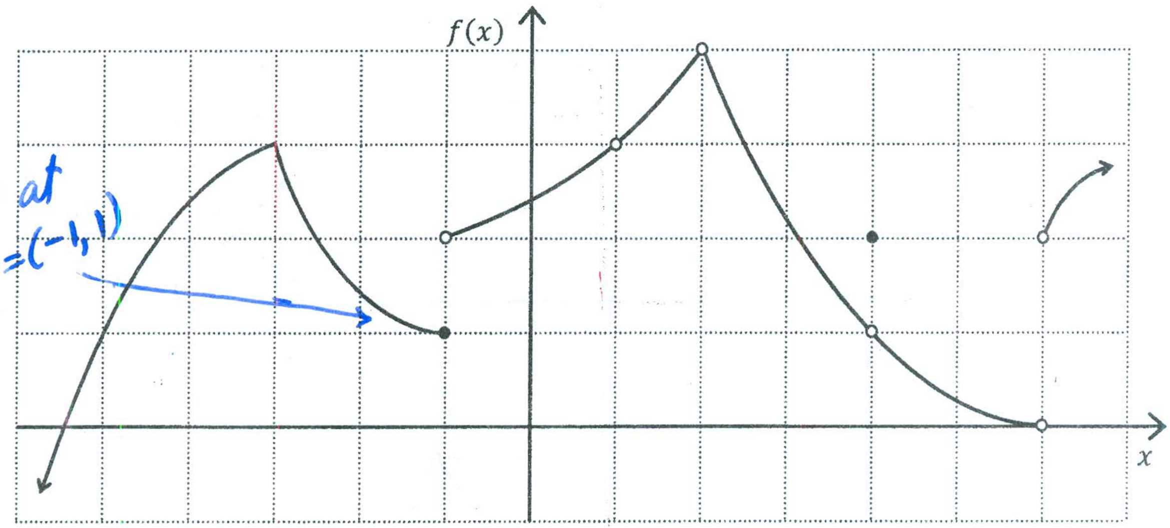
Now consider x values close to -1 but to right of -1.
When x is close to -1 but to right of -1, the y values are close to 2.

Symbol: $\lim_{x \to -1^+} f(x) = 2$

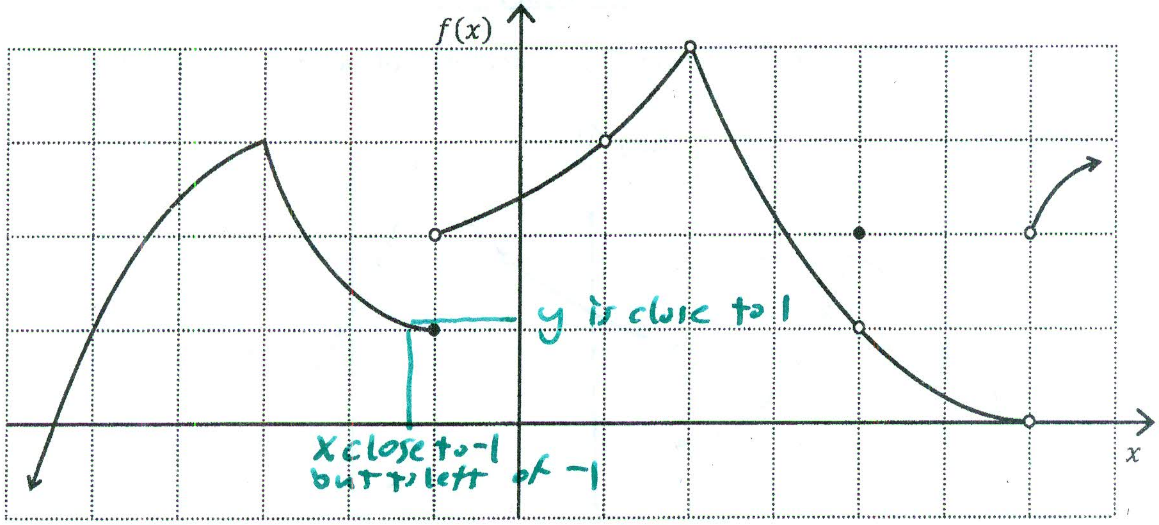
Class Drill 1: Limits

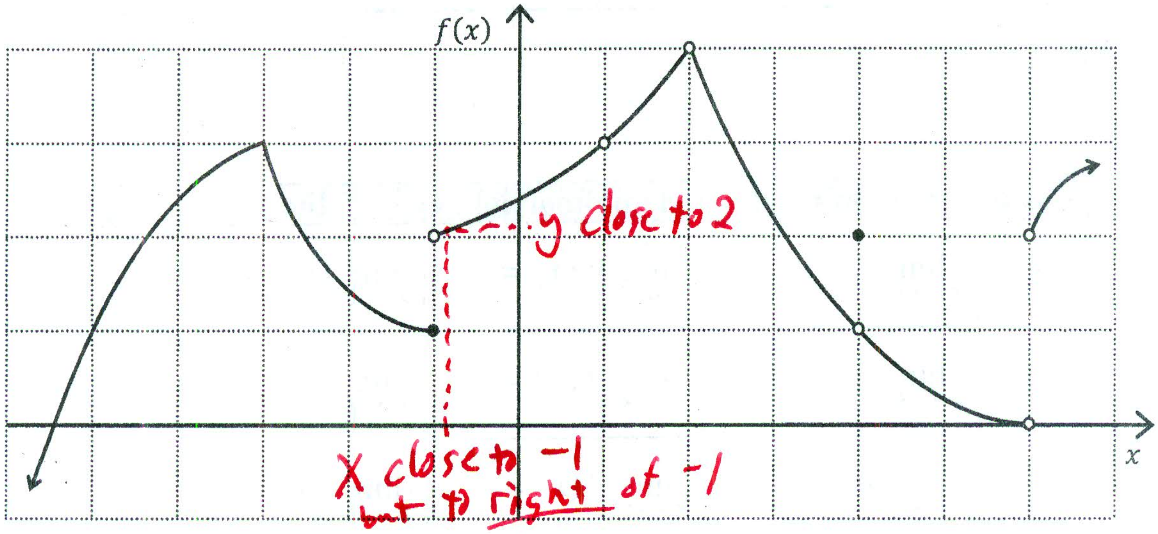
Use the graph to fill in the table. (Extra copies of the graph are on back.)

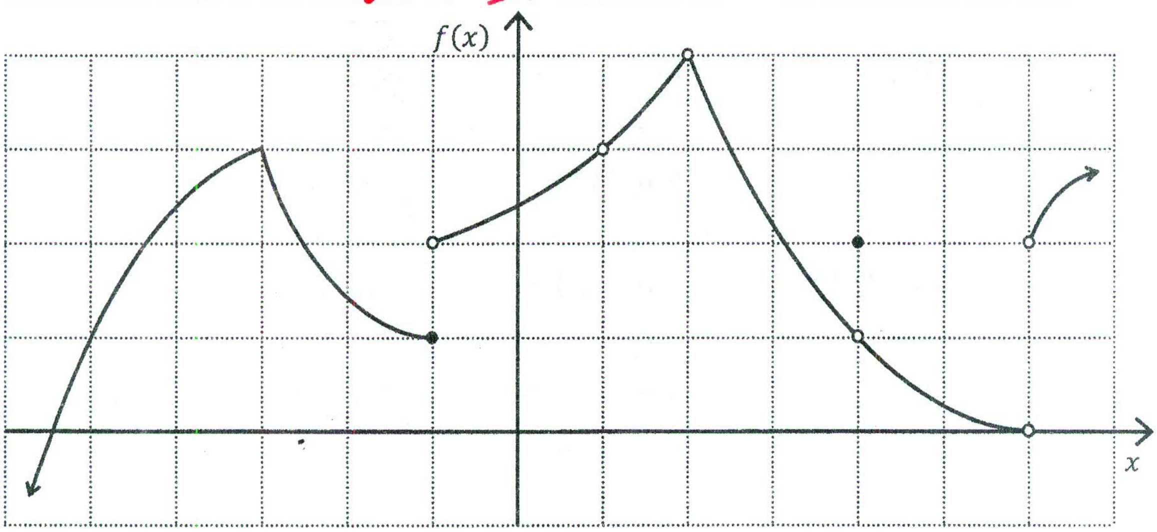
dot at $(x, y) = (-1, 1)$



x-value	limit from left	limit from right	limit	y-value
−5	$\lim_{x\to -5^-} f(x) =$	$\lim_{x\to -5^+} f(x) =$	$\lim_{x\to -5} f(x) =$	$f(-5) =$
−3	$\lim_{x\to -3^-} f(x) =$	$\lim_{x\to -3^+} f(x) =$	$\lim_{x\to -3} f(x) =$	$f(-3) =$
−1	$\lim_{x\to -1^-} f(x) = 1$	$\lim_{x\to -1^+} f(x) = 2$	$\lim_{x\to -1} f(x) =$ DNE	$f(-1) = 1$
1	$\lim_{x\to 1^-} f(x) = 3$	$\lim_{x\to 1^+} f(x) = 3$	$\lim_{x\to 1} f(x) = 3$	$f(1) =$ DNE
2	$\lim_{x\to 2^-} f(x) =$	$\lim_{x\to 2^+} f(x) =$	$\lim_{x\to 2} f(x) =$	$f(2) =$
4	$\lim_{x\to 4^-} f(x) = 1$	$\lim_{x\to 4^+} f(x) = 1$	$\lim_{x\to 4} f(x) = 1$	$f(4) = 2$
6	$\lim_{x\to 6^-} f(x) = 0$	$\lim_{x\to 6^+} f(x) = 2$	$\lim_{x\to 6} f(x) =$ DNE	$f(6) =$ DNE


f(x)
y is close to 1
x close to -1
but to left of -1
x


f(x)
y close to 2
x close to -1
but to right of -1
x


f(x)
x

Observe that for $\lim_{x \to -1} f(x)$, there is no y-value that will work. So $\lim_{x \to -1} f(x)$ DNE.

New definition of limit

Three-part test for the existence of a limit.

Test (a) $\lim_{x \to c^-} f(x)$ exists. (limit from left exists)

Test (b) $\lim_{x \to c^+} f(x)$ exists. (limit from right exists)

Test (c) The one-sided limits must match. If they do match, then the limit exists, and its value is equal to the values from (a), (b).

With this new definition of limit, go back and fill in some more of the table.

So far our examples have been of the type

Graph $\longrightarrow$ description of limit behavior.

Now do examples of the type

description of limit behavior $\longrightarrow$ draw a possible graph.

Exercise 3-1 #40 Draw a graph that satisfies all three properties

(1) $f(1) = 1$.

(2) $\lim\limits_{x \to 1^-} f(x) = 2$

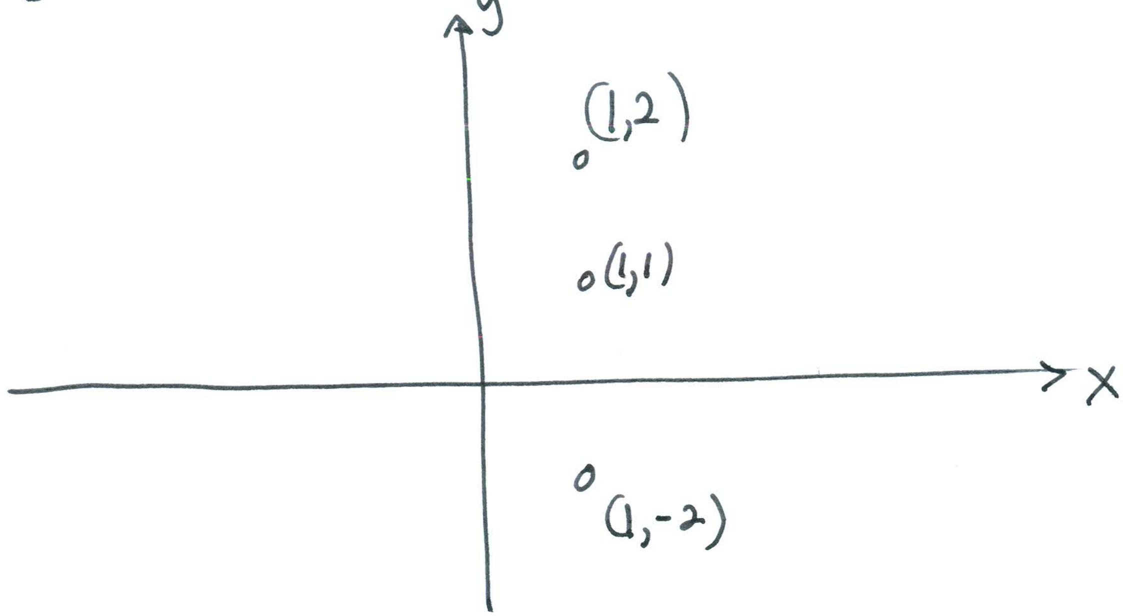
(3) $\lim\limits_{x \to 1^+} f(x) = -2$

Solution

The problem mentions three (x,y) locations. They are

$(x,y) = (1,1)$ $\quad$ $(x,y) = (1,2)$ $\quad$ $(x,y) = (1,-2)$

Start by plotting open circles at those three locations



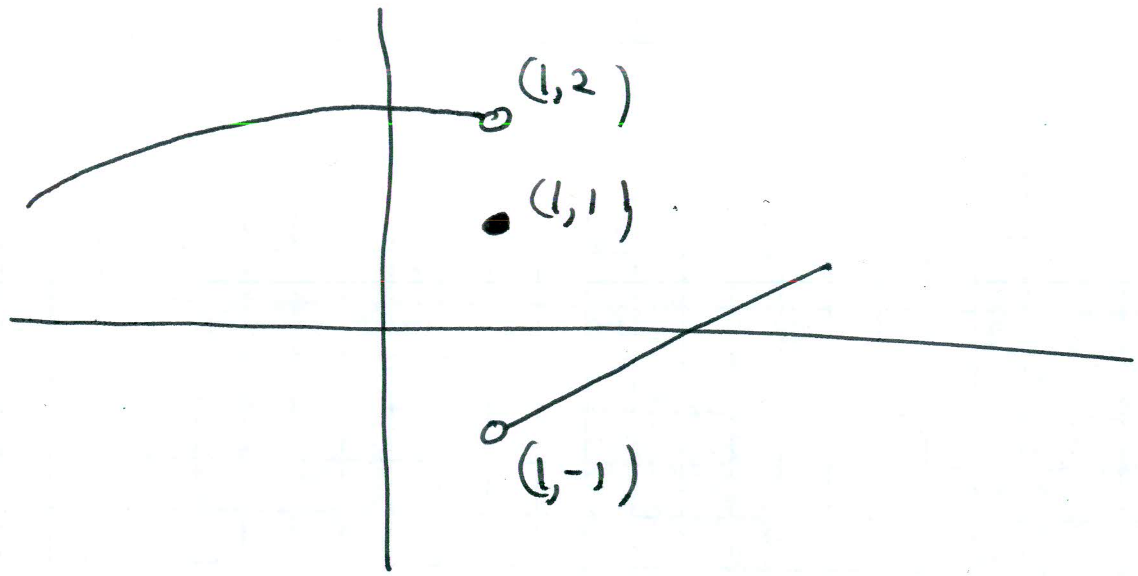
Now see what the problem statement says about each location.

The fact that $f(1)=1$ tells us that the circle at $(1,1)$ should be a dot, filled in.

The fact that $\lim_{x \to 1^-} f(x) = 2$ tells us that the graph should appear to be heading for $(x,y)=(1,2)$ from the left.

The fact that $\lim_{x \to 1^+} f(x) = -2$ tells us that the graph should appear to be heading for $(x,y)=(1,-2)$ from the <u>right</u>.

Update the graph with this information



Now discuss Analytic Approach to Limits.

(functions given by formulas, not by graphs.)

Basic tool that we will use: Reference 3 on page 4 of course packet.

Example $f(x) = -7x^2 + 13x - 29$

Find $\lim_{x \to 2} f(x)$.

Solution Observe that $f(x)$ is a polynomial, so we can use rule #3. (theorem 3)

$$\lim_{x \to 2} f(x) = f(2) \quad \text{(by ~~rule \#3~~ theorem 3)}$$

$$= -7(2)^2 + 13(2) - 29$$

exponent gets computed before multiplying

$$= -7(4) + 26 - 29$$

$$= -28 + 26 - 29$$

$$= -31$$

Example

Find $\lim\limits_{x \to 3} \sqrt{16 + x^2}$

Solution

$$\lim_{x \to 3} \sqrt{16 + x^2} = \sqrt{\lim_{x \to 3} 16 + x^2} \quad \text{by Theorem 2.8}$$

$$= \sqrt{16 + (3)^2} \quad \text{by Theorem 3}$$

$$= \sqrt{16 + 9}$$

$$\cancel{= \sqrt{16} + \sqrt{9}}$$

$$\cancel{= 4 + 3}$$

$$\cancel{= 7}$$

$$= \sqrt{25}$$

$$= 5$$

$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

Examples using Algebraic Simplification

let $f(x) = \dfrac{x^2+3x-10}{x-2} = \dfrac{(x-2)(x+5)}{(x-2)}$

(A) Find $f(c)$ for $c = 0, 1, 2, 3$

Solution

$f(0) = \dfrac{(0-2)(0+5)}{(0-2)} = \dfrac{\cancel{(-2)}(5)}{\cancel{(-2)}} = 5$

$f(1) = \dfrac{(1-2)(1+5)}{(1-2)} = \dfrac{\cancel{(-1)}(6)}{\cancel{(-1)}} = 6$

$f(2) = \dfrac{(2-2)(2+5)}{(2-2)} = \dfrac{(0)(7)}{(0)}$ cannot cancel the zeros!! $= \dfrac{0}{0}$ DNE !!

$f(3) = \dfrac{(3-2)(3+5)}{(3-2)} = \dfrac{\cancel{(1)}(8)}{\cancel{(1)}} = 8$

(B) Find $\lim_{x \to c} f(x)$ for $x = 0, 1, 2, 3$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\cancel{(x-2)}(x+5)}{\cancel{(x-2)}}$$

can cancel inside a limit (by the one-x-rule)

$$= \lim_{x\to 0} x+5$$

$$= 0 + 5$$ using theorem 3.

$$= 5$$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{\cancel{(x-2)}(x+5)}{\cancel{(x-2)}}$$

can cancel expressions inside a limit (by the one-x-rule)

$$= \lim_{x\to 1} x+5$$

$$= 1 + 5$$ using theorem 3

$$= 6$$

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+5)}{\cancel{(x-2)}}$$

can cancel terms inside a limit (by the one-x rule)

$$= \lim_{x\to 2} x+5$$

$$= 2+5$$ using theorem 3

$$= 7$$

This is very important

$f(2)$ DNE

but $\lim_{x\to 2} f(x) = 7$

Two very common mistakes

<u>one</u>

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{(x-2)(x+5)}{(x-2)}$$

$$= \frac{(2-2)(2+5)}{(2-2)}$$

Mistake here. there is no rule that allows us to plug in $x=2$

$$= \frac{0}{0} \quad \text{DNE}$$

<u>another</u>

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{(x-2)(x+5)}{(x-2)} = \frac{(\cancel{2-2})(2+5)}{(\cancel{2-2})} = 2+5 = 7$$

mistake

not true!!

Two mistakes!! Even worse, even though the final answer was correct.

Discuss why this example makes sense

$$f(x) = \frac{(x-2)(x+5)}{(x-2)} \qquad \text{let } g(x) = x+5$$

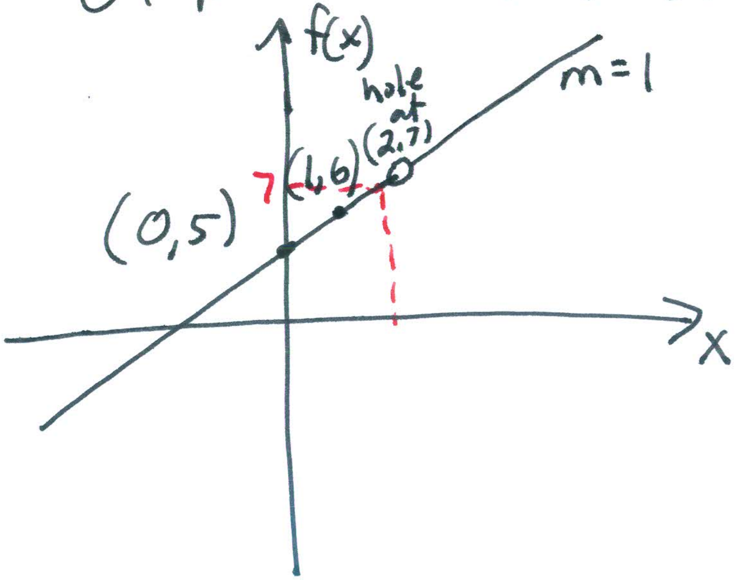
These functions are not the same because they have different domains.

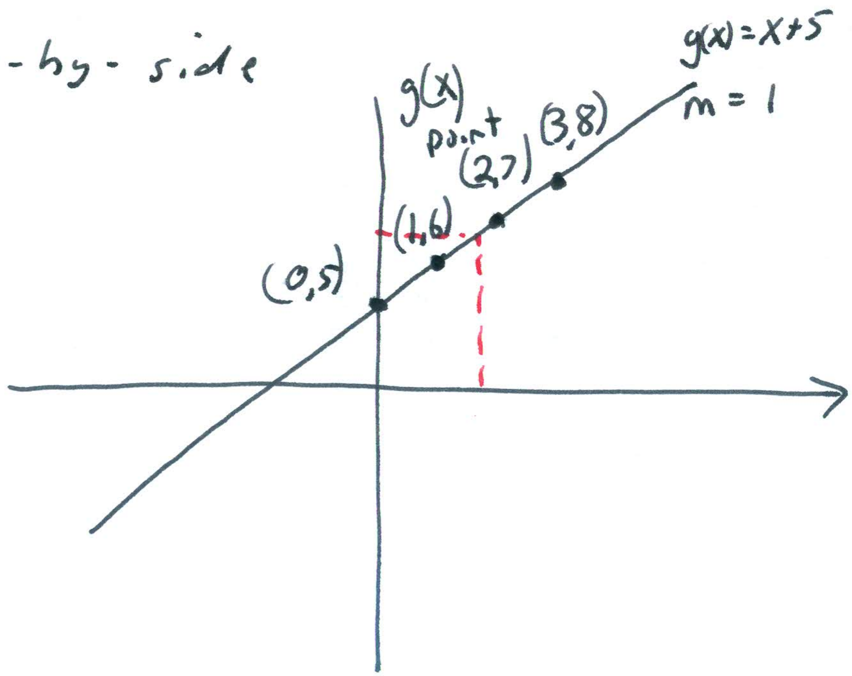
domain of f is all $x \neq 2$.

There was a typo here in class: I wrote $x=2$.

domain of g is all real numbers.

Graph the functions side-by-side





We see from graph of $f(x)$ that

$$f(2) \quad \text{DNE} \qquad (\text{hole})$$

For x close to 2, y is close to 7

$$\lim_{x \to 2} f(x) = 7$$

graph appears to be heading for

$$(x, y) = (2, 7)$$

<u>Notice</u> $f(2) \neq g(2)$

<u>but</u>

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} g(x)$$

This is the "one-x-rule" in Reference 3.