Day 6 is Monday, January 28, 2013

Section 3-4 The Derivative

Rates of Change

Series of examples involving the function

$$f(x) = -x^2 + 6x - 5 = -(x-1)(x-5)$$

standard form   factored

(A) draw the graph

Solution: From the standard form, we know the graph

$$-x^{2} + 6x - 5$$

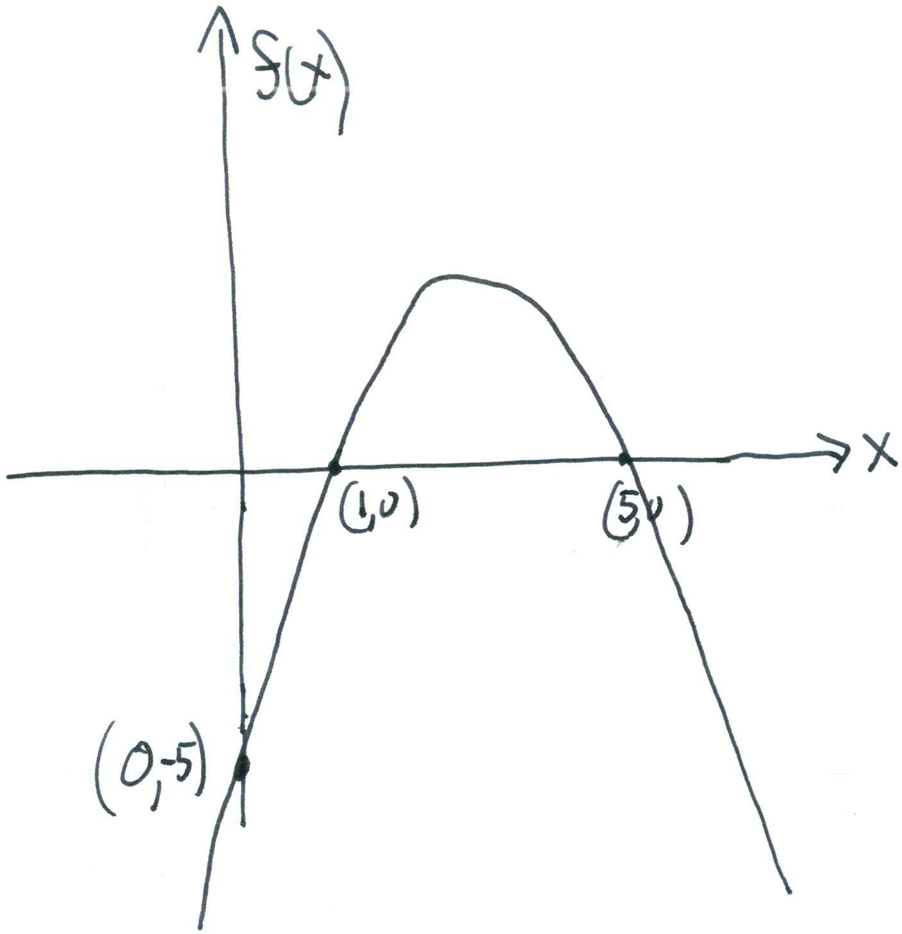
parabola

facing down   y-intercept at $(x,y) = (0,-5)$

From the factored form $-(x-1)(x-5)$

We know that x-intercepts at $(x,y) = (1,0)$ and $(5,0)$

With just that info, we can sketch a graph



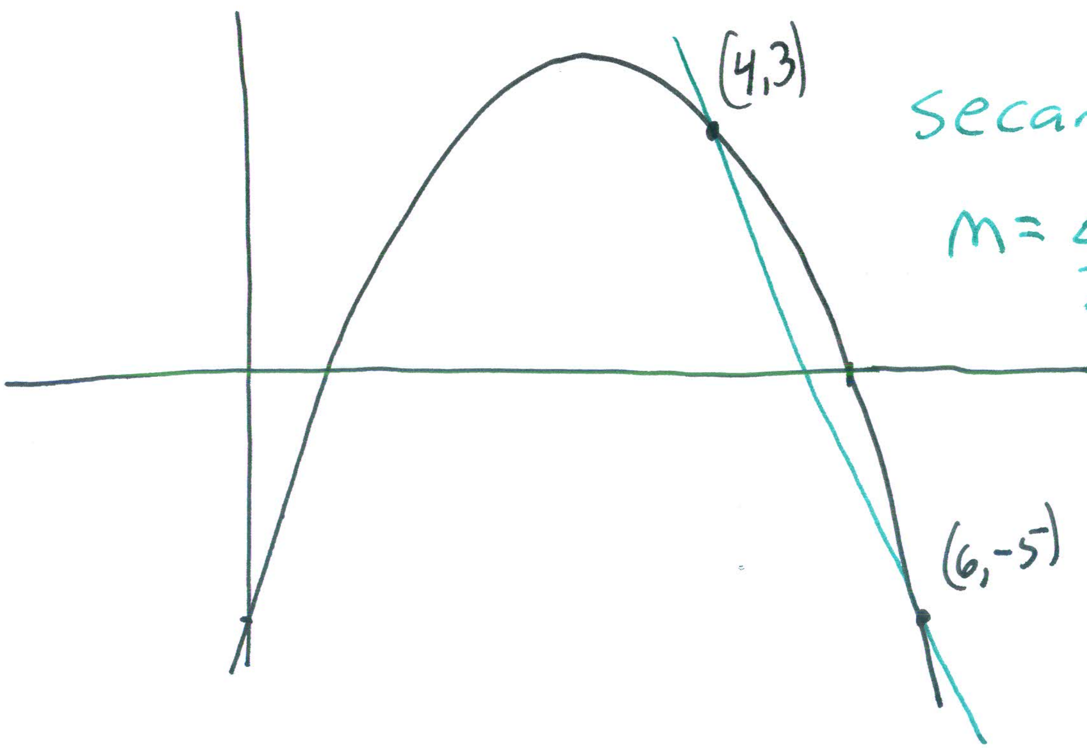
(B) draw the secant line that passes through the points $(x, y) = (4, f(4))$ and $(x, y) = (6, f(6))$

Solution: find $f(4)$ and $f(6)$

$$f(4) = -(4-1)(4-5) = -(3)(-1) = 3$$

$$f(6) = -(6-1)(6-5) = -(5)(1) = -5$$

the two points are $(x, y) = (4, 3)$ and $(x, y) = (6, -5)$



secant line

$$m = \frac{\Delta y}{\Delta x} = -4$$

(C) Find the slope of the secant line

Solution

$$m = \frac{\Delta y}{\Delta x} = \frac{(-5)-(3)}{6-4} = \frac{-8}{2} = -4$$

Introducing the Average Rate of Change

See Reference 4 Rates of Change on page 5 of the course packet.

we computed $m = \dfrac{f(6)-f(4)}{6-4} = \dfrac{(-5)-(3)}{6-4} = -4$

This is called "The Average Rate of Change of f from $x=4$ to $x=6$."

(D) New calculation: find the average rate of change of f from $x=4$ to $x=4+h$.

Solution We need to compute

$$m = \frac{f(4+h) - f(4)}{(4+h) - (4)}$$

we need to find the values $f(4)$ and $f(4+h)$

We already know $f(4) = 3$ from before.

To find $f(4+h)$, we go back to the original formula

$$f(x) = -x^2 + 6x - 5$$

$$f(\ ) = -(\ )^2 + 6(\ ) - 5 \quad \text{"empty version"}$$

Now put $4+h$ in each parentheses

$$f(4+h) = -(4+h)^2 + 6(4+h) - 5$$

$$= -((4+h)(4+h)) + 24 + 6h - 5$$

$$= -(4^2 + 4h + 4h + h^2) + 24 + 6h - 5$$

$$= -(4^2 + 8h + h^2) + 24 + 6h - 5$$

$$= -16 - 8h - h^2 + 24 + 6h - 5$$

$$f(4+h) = 3 - 2h - h^2$$

Now build the expression for m

$$m = \frac{f(4+h) - f(4)}{(4+h) - 4} = \frac{(3 - 2h - h^2) - (3)}{h}$$

$$= \frac{-2h - h^2}{h}$$

factor out an h in the numerator

$$= \frac{h(-2 - h)}{h}$$

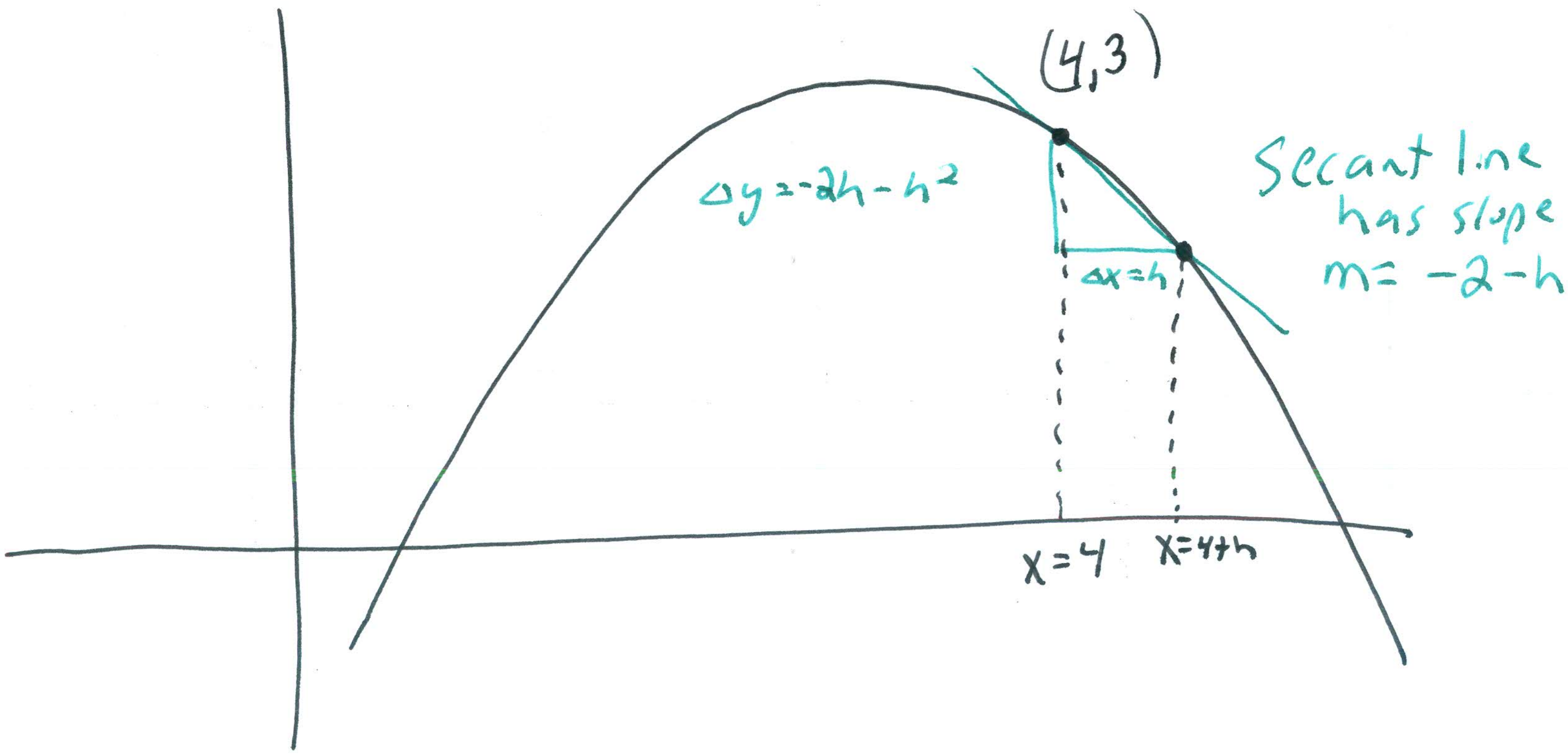
$$m = -2 - h$$

(E) Illustrate this quantity on the graph of f.

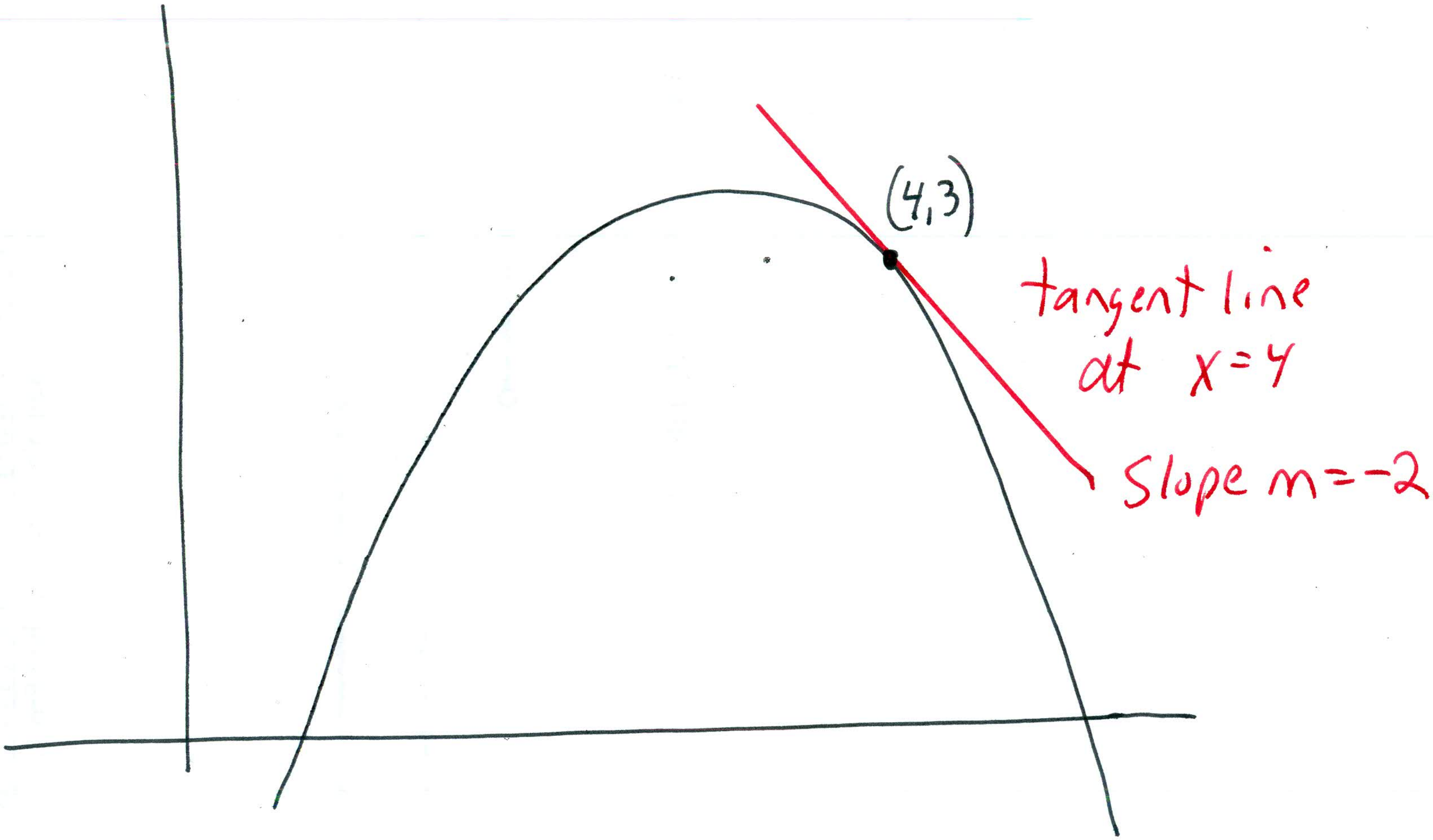
Solution:

m = slope of the secant line from $x=4$ to $x=4+h$

(just think of h as some small number)



(F) Draw the line tangent to the graph of f at $x=4$



(G) Find the slope of the tangent line.

Solution According to the Reference 4, we need to calculate

$$m = \underbrace{\lim_{h \to 0} \frac{\overbrace{f(4+h) - f(4)}^{\text{secant line slope}}}{\underbrace{h}}}_{}$$

tangent line slope

but we already know that $\frac{f(4+h) - f(4)}{h} = -2-h$

so

$$m = \lim_{h \to 0} -2-h$$

$= -2 - 0$ can substitute $h=0$

$= -2$

Slope of the line tangent at $X=4$