Day 9 is Monday, February 4, 2013

Today: Start Section 3-5

Basic Differentiation Properties

In Section 3-4, we learned about the Definition of the derivative. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

We computed $f'(x)$ by using this definition.
(we organized our solutions using "4-step method")

Step 1 find $f(x+h)$

Step 2 find $f(x+h) - f(x)$

Step 3 find $\frac{f(x+h) - f(x)}{h}$

Step 4 find $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Starting Now, in Section 3-5, we will learn Rules for Taking Derivatives. We won't use the 4-step method,

Notation for the Derivative

Notation without variables:

function: f

the derivative: f'

Notation using variables:

function: $f(x)$

the derivative: $f'(x)$ or $\frac{df}{dx}$ or $\frac{df(x)}{dx}$

Syntax:

$$\underbrace{\underbrace{\frac{d}{dx}}_{\text{this tells us to take the derivative}} \underbrace{f(x)}_{\text{the function}}}$$

the whole symbol $\frac{df(x)}{dx}$ stands for the result, the derivative.

Our First Derivative Rule:

The Constant function Rule

- If $f(x)$ is a constant function, then $f'(x)$ is zero.
- If $f(x) = c$ then $f'(x) = 0$
- $\dfrac{dc}{dx} = 0$

Example

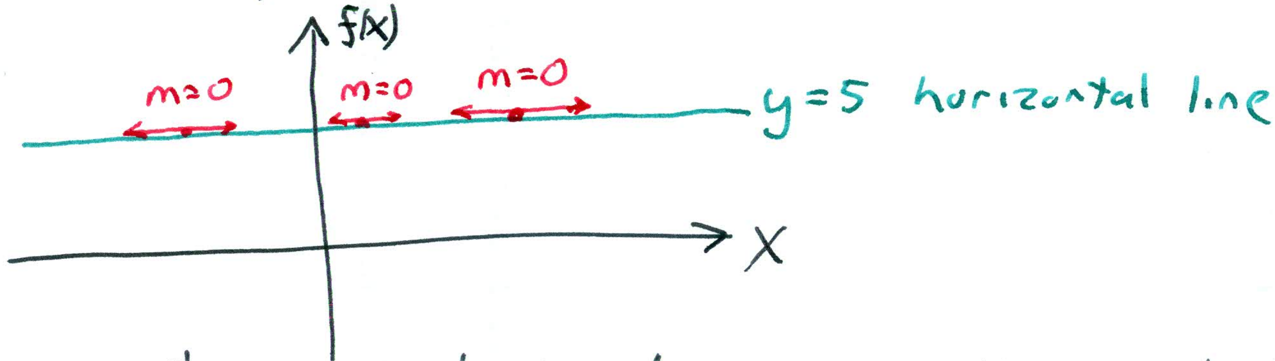
If $f(x) = 5$ then $f'(x) = 0$

Equivalently: $\dfrac{d5}{dx} = 0$

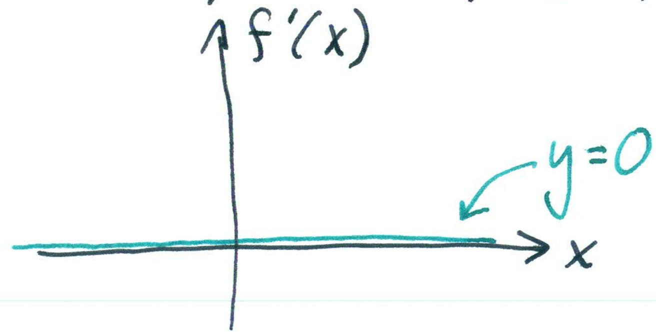
Why does this rule make sense?

from Reference 4 on pages of course packet, we know that $f'(a)$ represents the number m that is the slope of the line that is tangent to graph of f at $x=a$.

Consider the graph of the constant function $f(x)=5$



Imagine tangent lines drawn on the graph of $f(x)$. Every tangent line has slope $m=0$. So the value of $f'(x)$ must always be 0.



Review Terminology

Power function is a function of the form

$$f(x) = x^{p}$$ real number constant exponent

variable base

Examples Which of these are power functions?

$f(x) = x^3$ yes $p = 3$

$f(x) = x^{-3}$ yes $p = -3$

$f(x) = \dfrac{1}{x}$ yes. rewrite $f(x) = x^{-1}$ $p = -1$

$f(x) = x^{\pi}$ yes $p = \pi$ a real number

$f(x) = \pi^{x}$ no. (Variable exponent, constant base)

$f(x) = 1$ yes. rewrite $1 = x^0$ $p = 0$

$f(x) = \sqrt{x}$ yes: rewrite $x^{1/2}$ $p = \frac{1}{2}$

The Power Rule

Used for taking the derivative of power functions.

two equation form: If $f(x) = x^n$ then $f'(x) = n \cdot x^{n-1}$

single equation form: $\frac{d x^n}{dx} = n \cdot x^{n-1}$

Examples

If $f(x) = x^3$ ← $n=3$ then $f'(x) = 3x^{3-1}$ power rule

$= 3x^2$ simplified

~~Example~~

If $f(x) = \frac{1}{x^3}$ then $f'(x) = -3 \cdot x^{-3-1} = -3 \cdot x^{-4}$

dirty work: rewrite $f(x) = \frac{1}{x^3} = x^{-3}$ ← $n=-3$

$= -3 \cdot \frac{1}{x^4}$

$= \frac{-3}{x^4}$

Rule for our class:

When presenting the solution to ~~the~~ a problem in this class, your ~~~~ final version of the answer must not have any negative exponents.

Why?!?

Our last example: we found $f'(x) = -3X^{-4} = \frac{-3}{X^4}$

If I say: $f'(x) = -3X^{-4}$ and ask you to substitute in $x = -2$, Most students are unable to do that.

$$f'(-2) = -3(-2)^{-4} = ???$$

But if I say $f'(x) = \dfrac{-3}{x^4}$ and ask you to substitute in $x = -2$, a lot more students <u>can</u> do that.

$$f'(-2) = -\frac{3}{(-2)^4} = -\frac{3}{16}$$

<u>Example</u> If $f(x) = \dfrac{1}{x}$ find $f'(x)$.

Solution

Rewrite $f(x) = \dfrac{1}{x} = x^{-1}$ ← $n = -1$

So $f'(x) = (-1)x^{-1-1}$ power rule

$= (-1)x^{-2}$ simplified

$= (-1)\dfrac{1}{x^2}$ rewritten with pos. exponent

$= -\dfrac{1}{x^2}$ this agrees with the answer that we got in class on Tues Jan 29 using the Four-Step Method.

Harder example

Let $f(x) = \sqrt{x}$ find $f'(x)$

Solution: Start by rewriting $f(x)$ as a power function.

$$f(x) = \sqrt{x} = x^{1/2} \leftarrow n = \tfrac{1}{2}$$

$$f'(x) = \left(\frac{1}{2}\right) x^{\frac{1}{2}-1}$$ power rule

$$= \left(\frac{1}{2}\right) \cdot x^{-\frac{1}{2}}$$ simplified

$$= \left(\frac{1}{2}\right) \cdot \frac{1}{x^{1/2}}$$ eliminating negative exponent used fact that $a^{-b} = \frac{1}{a^b}$

$$= \left(\frac{1}{2}\right) \cdot \frac{1}{\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

If $f(x) = 1$ find $f'(x)$

Solution: rewrite $f(x)$ as a power function

$$f(x) = 1 = x^0 \leftarrow n = 0$$

$$\text{So } f'(x) = 0 \cdot x^{0-1} \quad \text{power rule}$$

$$= 0 \cdot x^{-1} \quad \text{Simplified}$$

$$= 0 \quad \text{Simplified}$$

This makes sense, because $f(x) = 1$ is a constant function, so $f'(x)$ should be 0.

If $f(x) = x$, find $f'(x)$

Solution:

Rewrite f as a power function $f(x) = x = x^{1}$ $n=1$

So $f'(x) = (1) \cdot x^{1-1}$ power rule

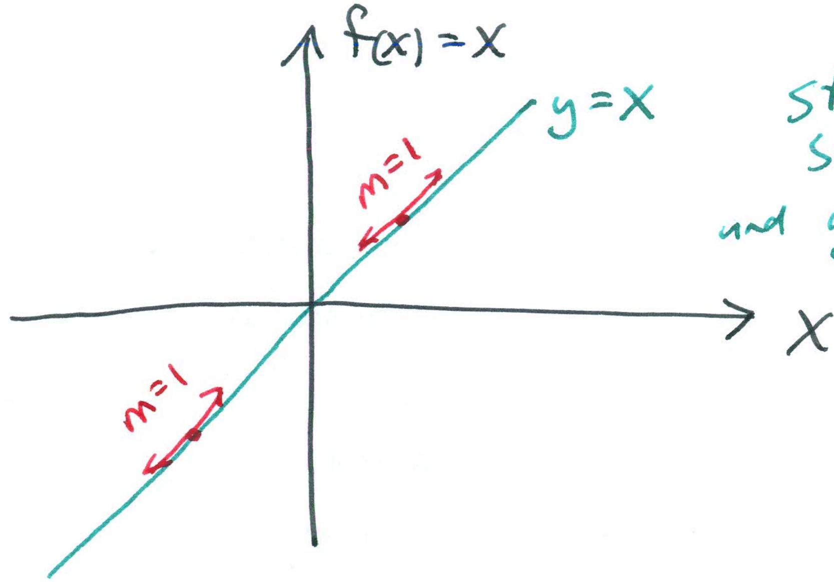
$= (1) \cdot x^{0}$

$= (1) \cdot (1)$ simplify

$= 1$

Does this make sense?

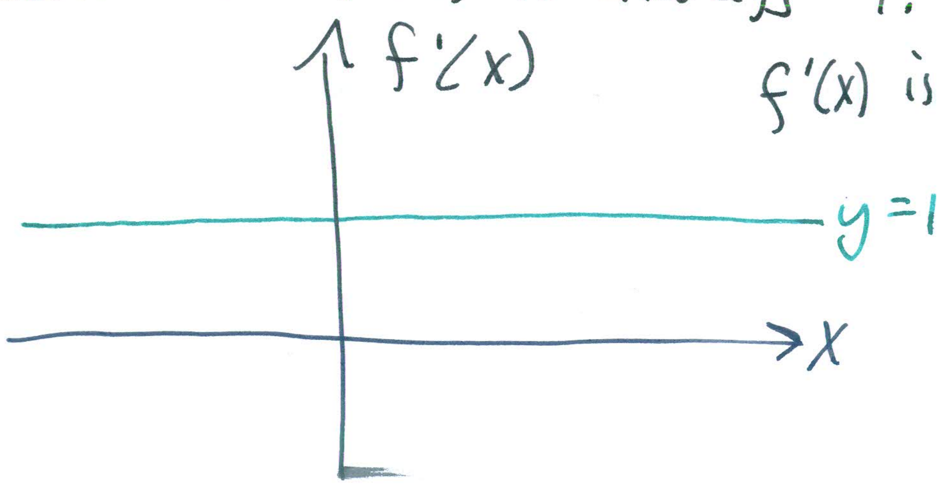
Consider the graph of $f(x) = x$.



straight line with slope $m=1$ and y-intercept $b=0$

Consider the tangent lines. They <u>all</u> have slope $m=1$.

So the value of $f'(x)$ is always 1.



$f'(x)$ is the constant function $f'(x) = 1$.