Day 10 is Tuesday, February 5, 2013

Quiz 3 Thursday

Practice writing out solutions to Examples + Match Problems

You will need to show steps clearly, with proper notation.

New Rule (a combination of two rules in the book)

The Sum Rule and Constant Multiple Rule

Single ~~Two~~ equation form

$$\frac{d}{dx}\Big(\underbrace{a}_{\text{constant multiple}}\cdot\underbrace{f(x)}_{\text{function}}+\underbrace{b}_{\text{constant multiple}}\cdot\underbrace{g(x)}_{\text{function}}\Big)=a\cdot\frac{df(x)}{dx}+b\cdot\frac{dg(x)}{dx}$$

the constant multiples come right through the derivative symbol and become constant multiples in front.

Examples

#1 (A) find derivative of $f(x) = -3x^2 + 5x - 7$

Solution Notice that I write out the steps in detail

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left(-3 x^2 + 5 x - 7 \right)$$

identify constant multiples

$$= -3 \frac{d}{dx} x^{2} + 5 \frac{d}{dx} x - \frac{d}{dx} 7$$

n=2

Sum + constant multiple rule

$$= -3\left(2 \cdot x^{2-1}\right) + 5(1) - (0)$$

Power Rule

from yesterday power rule

from yesterday

$$\boxed{f'(x) = -6x + 5}$$

Notice that this agrees with ~~the~~ the result ~~that~~ that we got last Tuesday Jan 29 in class using the definition of the derivative.

(B) Find x-coordinates of all points on graph of $f(x) = -3x^2+5x-7$ that have tangent lines with a slope of 11.

Solution

We are told that $m = 11$.

But m is obtained from $f'(x)$.

So we are being told that $f'(x) = 11$

and we are being asked for x.

Strategy: Set $f'(x) = 11$ and solve for x.

$$-6x + 5 = 11 \quad \text{solve for } x$$

$$-6x + 5 - 11 = 0$$

$$-6x - 6 = 0$$

$$-6x = 6$$

$$x = -1$$

More Difficult derivative problems

#2 $f(x) = \frac{23x^6}{3} + \frac{19x}{5} + \frac{17}{7x} + \frac{13}{11x^6}$ Find $f'(x)$.

Solution

Start by rewriting f in the form

constants·power functions.

$$f(x) = \left(\frac{23}{3}\right)x^6 + \left(\frac{19}{5}\right)x + \left(\frac{17}{7}\right)\left(\frac{1}{x}\right) + \left(\frac{13}{11}\right)\left(\frac{1}{x^6}\right)$$

$$= \left(\frac{23}{3}\right)x^6 + \left(\frac{19}{5}\right)x + \left(\frac{17}{7}\right)x^{-1} + \left(\frac{13}{11}\right)x^{-6}$$

Now take the derivative

$$f'(x) = \frac{d}{dx}\left(\left(\frac{23}{3}\right)x^6 + \left(\frac{19}{5}\right)x + \left(\frac{17}{7}\right)x^{-1} + \left(\frac{13}{11}\right)x^{-6} \right)$$

identify constant multiples

$$= \frac{23}{3}\frac{d}{dx}x^6 + \left(\frac{19}{5}\right)\left(\frac{d}{dx}x\right) + \left(\frac{17}{7}\right)\left(\frac{d}{dx}x^{-1}\right) + \left(\frac{13}{11}\right)\left(\frac{d}{dx}x^{-6}\right)$$

Sum and constant multiple rule

$$= \left(\frac{23}{3}\right)\left(6x^5\right) + \left(\frac{19}{5}\right)\left(1\right) + \left(\frac{17}{7}\right)\left((-1)x^{-2}\right) + \left(\frac{13}{11}\right)\left((-6)x^{-7}\right)$$

power rule

$$= 46x^5 + \frac{19}{5} - \frac{17}{7x^2} - \frac{78}{11x^7}$$

Simplify & get rid of negative exponents.

#3 Same idea, but more difficult

$$f(x) = \frac{2\sqrt[5]{x}}{7} - \frac{3}{11x^{2/5}} \quad \text{find } f'(x).$$

Solution

Start by rewriting $f(x)$ as constants * power functions

$$f(x) = \left(\frac{2}{7}\right)x^{\frac{1}{5}} - \left(\frac{3}{11}\right)x^{-\frac{2}{5}}$$

Now find $f'(x)$

$$f'(x) = \left(\frac{2}{7}\right)\frac{d}{dx}x^{1/5} - \left(\frac{3}{11}\right)\frac{d}{dx}x^{-2/5}$$

$n = \frac{1}{5}$ $\quad$ $n = -\frac{2}{5}$

incorporated the Sum rule + constant multiple rule into the 1st step

$$= \left(\frac{2}{7}\right)\left(\left(\frac{1}{5}\right)\cdot x^{\frac{1}{5}-1}\right) - \left(\frac{3}{11}\right)\left(\left(\frac{-2}{5}\right)x^{-\frac{2}{5}-1}\right)$$

$$= \left(\frac{2}{7\cdot 5}\right) x^{-\frac{4}{5}} + \left(\frac{6}{55}\right) x^{-7/5}$$

simplified

Still have to get rid of negative exponents!

$$= \left(\frac{2}{35}\right)\frac{1}{x^{4/5}} + \left(\frac{6}{55}\right)\frac{1}{x^{7/5}}$$

$$f'(x) = \frac{2}{35x^{4/5}} + \frac{6}{55x^{7/5}}$$

cleaned up.

Note! fractional exponents are good: $x^{4/5}$

Crazy radicals are bad: $\sqrt[5]{x^4}$

#4 Trick problem

$$f(x) = \frac{2x^5 - 4x^3 + 2x}{x^3} \qquad \text{find } f'(x).$$

Solution: (the trick) separate into three fractions and rewrite as constants & power functions

$$f(x) = \frac{2x^5}{x^3} - \frac{4x^3}{x^3} + \frac{2x}{x^3}$$

$$= 2x^2 - 4 + \frac{2}{x^2}$$

$$= 2x^2 - 4 + 2x^{-2}$$

$$f'(x) = 2\frac{d}{dx}x^2 - \frac{d}{dx}4 + 2\frac{d}{dx}x^{-2}$$

$$= 2(2x) - 0 + 2((-2)x^{-2-1})$$

$$= 4x + 0 - 4x^{-3}$$

$$f'(x) = 4x - \frac{4}{x^3}$$

Special Topics Problems

Tangent line Problems for

$$f(x) = x^3 - 9x^2 + 15x + 25$$

(A) Find the slope of the line tangent to the graph of f at $x=2$.

Solution We are asked to find $m = f'(2)$.

Strategy: find $f'(x)$

Substitute $x=2$ into $f'(x)$ to get $m=f'(2)$.

$$f'(x) = \frac{d}{dx}\left(x^3 - 9x^2 + 15x + 25\right)$$

$$= \left(\frac{d}{dx}x^3\right) - 9\left(\frac{d}{dx}x^2\right) + 15\left(\frac{d}{dx}x\right) + \left(\frac{d}{dx}25\right)$$

$$= \left(3x^2\right) - 9\left(2x\right) + 15\left(1\right) + \left(0\right)$$

$$= 3x^2 - 18x + 15$$

note: the derivation is a function

$$m = f'(2) = 3(2)^2 - 18(2) + 15$$

$$= 3 \cdot 4 - 36 + 15$$

$$= -9 = m$$

(B) Find x-coordinates of all points on graph that have horizontal tangent lines.

Solution: Set $f'(x) = 0$ and solve for x.

$$3x^2 - 18x + 15 = 0$$

$$3(x^2 - 6x + 5) = 0$$

$$3(x-1)(x-5) = 0$$

Solutions: $x = 1$ and $x = 5$

(C) Find x-coordinates of all points where there is a tangent line with slope 36.

Solution: Set ~~ans~~ $f'(x) = 36$, solve for x.

$$3x^2 - 18x + 15 = 36$$

subtract 36 from both sides

$$3x^2 - 18x - 21 = 0$$

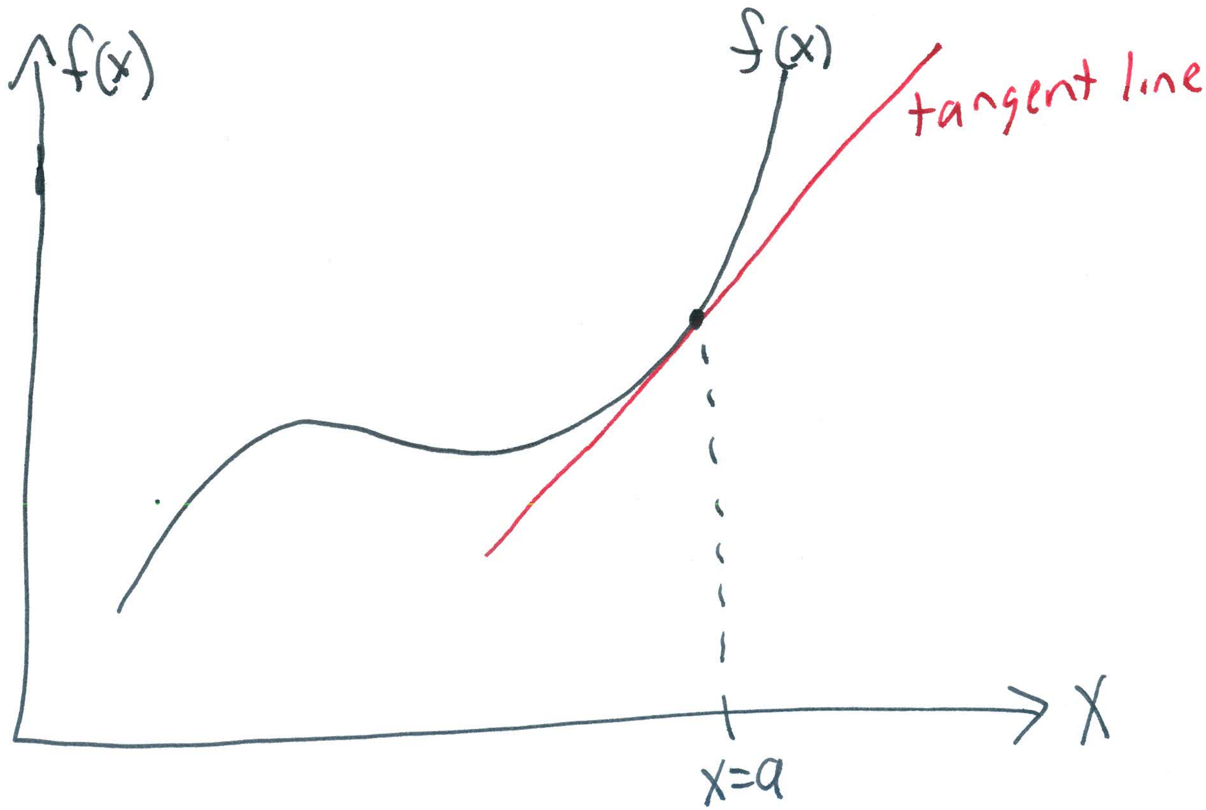
$$3(x^2 - 6x - 7) = 0$$

$$3(x + 1)(x - 7) = 0$$

Solutions: $x = -1$ and $x = 7$

Discuss the Equation For The Tangent Line

Goal: Given a function $f(x)$, write down the general form for the equation of the line tangent to graph of $f(x)$ at $x=a$.



Things that we know about the tangent line.

#1 The tangent line touches the graph at the point where $x=a$.

The y-coordinate of that point is $f(a)$

So the point of tangency is

$$(x,y)=(a,f(a))$$

This point is on the line.

#2 The slope of the tangent line is

$$m=f'(a)$$

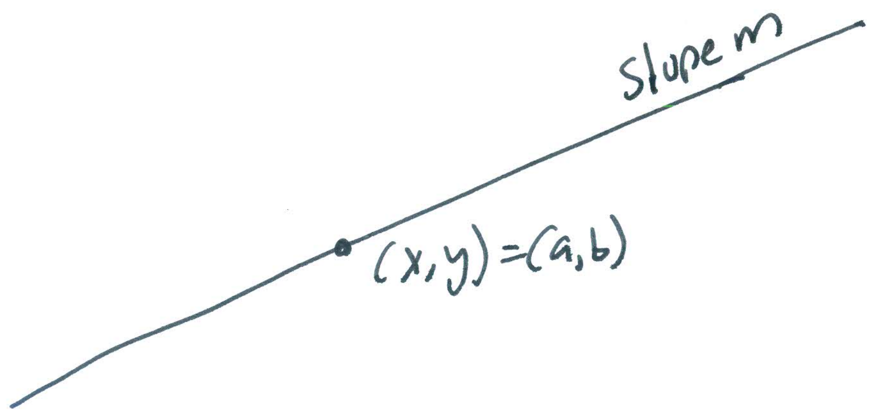
Review: Point Slope form of equation for a line.

Suppose that you know that a line

- contains the point $(x,y) = (a,b)$
- has slope m

The point slope form of the equation for the line is

$$(y - b) = m(x - a)$$



Apply this idea to the tangent line

- tangent line passes through the point

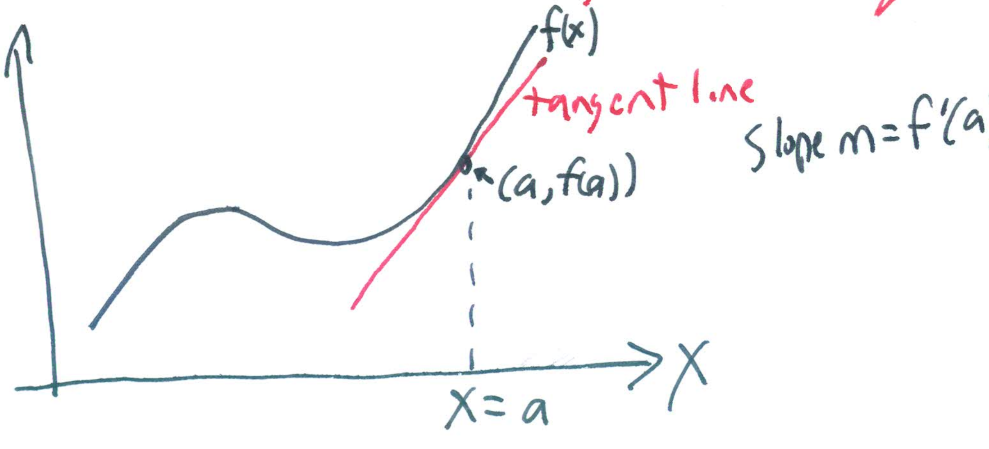
$$(x, y) = (a, f(a))$$

- tangent line has slope $m = f'(a)$

Point-Slope form of the equation for tangent line

$$(y - f(a)) = f'(a)(x - a)$$

general form of the tangent line equation



Return to our function $f(x) = x^3 - 9x^2 + 15x + 25$

Question (D) Find equation of the line tangent to graph of $f(x)$ at $x = 2$.

Solution

We need to build the equation

$$(y - f(a)) = f'(a)(x - a)$$

Get Parts

$a = 2$ (the x-coord of point of tangency)

$$f(a) = f(2) = 2^3 - 9(2)^2 + 15(2) + 25$$
$$= 8 - 36 + 30 + 25$$
$$= 27$$ (y-coordinate of point of tangency)

$f'(x) = 3x^2 - 18x + 15$ (we did this before)

$f'(a) = f'(2) = 3(2)^2 - 18(2) + 15$

$= -9$ we did this before

Assemble the equation

$$(y - 27) = -9(x - 2)$$

convert to slope intercept form

$$y - 27 = -9x + 18$$

$$y = -9x + 45$$