Day 15 is Monday, February 18, 2013

One more example involving the Derivative of an Exponential Function

Let $f(x) = 11e^{(x)} + 23x$

Find the equation of the line that is tangent to the graph of $f(x)$ at $x=0$.

Solution

We need to build the tangent line equation

$$(y - f(a)) = f'(a)\cdot(x-a)$$

Get Parts

$a=0$ (this is the x-coordinate of the point of tangency)

$f(a) = f(0) = 11e^{(0)} + 23(0) = 11\cdot(1) + 0 = 11$

(this is the y-coordinate of the point of tangency)

$$f'(x) = \frac{d}{dx}\left(11e^{(x)} + 23x\right)$$

$$= 11\frac{d}{dx}e^{(x)} + 23\frac{d}{dx}x$$

$$= 11(e^{x}) + 23(1)$$

$$= 11e^{x} + 23$$

$$f'(a) = f'(0) = 11 \cdot e^{(0)} + 23 = 11(1) + 23 = 34$$

(this is the slope m of the tangent line)

Substitute Parts into the Equation

$$(y - 11) = 34(x - 0)$$

Convert to Slope Intercept form

$$y - 11 = 34x$$

$$\boxed{y = 34x + 11}$$ Tangent line equation

Derivatives of Logarithmic Functions

Two New Derivative Rules

Logarithmic Function Rule #1

If $f(x) = \ln(x)$ then $f'(x) = \frac{1}{x}$ two equation form

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$ single equation form

Logarithmic function Rule #2

If $f(x) = \log_b(x)$ then $f'(x) = \frac{1}{x \cdot \ln(b)}$ two equation form

$$\frac{d}{dx}\log_b(x) = \frac{1}{x\ln(b)}$$ single equation form

Examples Find Derivatives of the following functions

(A) $f(x) = 12\ln(x)$

Solution $f'(x) = \frac{d}{dx} 12 \ln(x) = 12\frac{d}{dx}\ln(x) = 12\left(\frac{1}{x}\right) = \frac{12}{x}$

Log function Rule #1

(B) $f(x) = 12\log_{13}(x)$

Solution $f'(x) = 12\frac{d}{dx}\log_{13}(x) = 12\left(\frac{1}{x\ln(13)}\right) = \frac{12}{x\ln(13)}$

Log rule #2

(C) $f(x) = 12\log(x)$

Solution

Common convention $\log(x)$ always means $\log_{10}(x)$

except that it doesn't always mean that!!

In this book, $\log(x)$ means $\log_{10}(x)$

But in some books, and in some computer programs, and on some web sites, $\log(x)$ means $\ln(x)$

So we rewrite $f(x)$ as

$$f(x) = 12\log(x) = 12\log_{10}(x)$$

$$\text{So } f'(x) = 12\frac{d}{dx}\log_{10}(x) = 12\left(\frac{1}{x\ln(10)}\right) = \frac{12}{x\ln(10)}$$

used Logarithm rule #2 with base b=10

(D) $f(x) = 12\ln(13)$

$$f'(x) = \frac{d}{dx}\left(12\ln(13)\right) = 0$$

constant function

(E) $f(x) = 12\ln(13x)$

Solution: Our rules #1 + #2 don't allow anything inside the log function except just x

So we must rewrite our function

log rule: $\log(a \cdot b) = \log(a) + \log(b)$

$\ln(a \cdot b) = \ln(a) + \ln(b)$

use this to rewrite $f(x)$

$$f(x) = 12\ln(13x) = 12\left[\ln(13) + \ln(x)\right]$$

$$= 12\ln(13) + 12\ln(x)$$

Now take derivative

$$f'(x) = \underbrace{\frac{d}{dx} 12\ln(13)}_{\text{example (D)}} + \underbrace{\frac{d}{dx} 12\ln(x)}_{\text{example (A)}}$$

$$= 0 + \frac{12}{x}$$

$$= \frac{12}{x}$$

Class Drill #6 on page 24

Work on parts (A), (B), (C)