Day 16 is Tues Feb 19, 2013

Pick up your graded work.

Resume working on Class Drill 6.

Class Drill 6 Derivatives of Functions Containing Logarithms

(A) Let $f(x) = 12\ln\left(\frac{13}{x}\right)$. Find $f'(x)$. Hint: Start by rewriting f using a rule of logarithms.

rule of logs: $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

Rewrite $f(x) = 12(\ln(13) - \ln(x)) = 12\ln(13) - 12\ln(x)$

Now take derivative

$f'(x) = \frac{d}{dx}\underbrace{12\ln(13)}_{\text{constant}} - \frac{d}{dx}12\ln(x) = 0 - 12\frac{d\ln(x)}{dx} = -12\left(\frac{1}{x}\right)$

$= -\frac{12}{x}$

(B) Let $f(x) = 12\ln(x^{13})$. Find $f'(x)$. Hint: Start by rewriting f using a rule of logarithms.

rule of logs: $\ln(a^b) = b\ln(a)$

Rewrite $f(x) = 12 \cdot 13 \cdot \ln(x)$

Now take derivative

logarithm rule #1

$f'(x) = 12 \cdot 13 \cdot \frac{d}{dx}\ln(x) = 12 \cdot 13 \cdot \left(\frac{1}{x}\right) = \frac{156}{x}$

(C) Let $f(x) = 12x\ln(13)$. Find $f'(x)$.

Rewrite $f(x)$ with constant multiples in front

$f(x) = 12\ln(13) \cdot x$

Now take derivative

power rule

$f'(x) = \frac{d}{dx}(12\ln(13)) \cdot x = 12\ln(13)\frac{dx}{dx} = 12\ln(13) \cdot 1 = \boxed{12\ln(13)}$

(D)The goal is to find the equation of the line tangent to the graph of the function

$$f(x) = 5 + \ln(x^3)$$

at the point where $x = e^2$.

Remember that the approach is to build the general form of the equation for the tangent line (in point-slope form):

$$(y - f(a)) = f'(a) \cdot (x - a)$$

Question (D) continues on back. ➔

Get Parts

Identify the number a. $a = e^2$ this is the x-coord of the point of tangency

Find $f(a)$.

$$f(a) = f(e^2) = 5 + \ln((e^2)^3) = 5 + \ln(e^6) = 5+6 = 11$$

this is the y-coord of point of tangency

Find $f'(x)$. Hint: Start by rewriting f using a rule of logarithms.

rewrite $f(x) = 5 + \ln(x^3) = 5 + 3\ln(x)$

↑ rule of logs

Now take derivative

$$f'(x) = \frac{d}{dx}(5 + 3\ln(x)) = \frac{d}{dx}5 + 3\frac{d}{dx}\ln(x) = 0 + 3\left(\frac{1}{x}\right) = \frac{3}{x}$$

constant function

Find $f'(a)$.

$$f'(a) = f'(e^2) = \frac{3}{e^2}$$

this number is the slope m of the tangent line

Substitute Parts Into the General Tangent Line Equation

$$(y - 11) = \frac{3}{e^2}(x - e^2)$$

Convert the Equation to Slope Intercept Form

$$y - 11 = \frac{3}{e^2}(x - e^2)$$

$$= \left(\frac{3}{e^2}\right)x - \frac{3}{e^2} \cdot e^2$$

these cancel

$$\boxed{y = \left(\frac{3}{e^2}\right)x + 8}$$

Section 4-3 Derivatives of Products & Quotients

The Product Rule used for taking the derivative of a product of functions: $f(x) \cdot g(x)$

~~The obvious thing would be~~

$$\cancel{\frac{d}{dx}\left(f(x) \cdot g(x)\right) = \frac{df(x)}{dx} \cdot \frac{dg(x)}{dx}}$$

The obvious thing is wrong!

The Product Rule is not obvious, and it is harder

$$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Example#1 find derivative of $(-3x^2+5x-7)(3x-2)$ using the product rule. DONT SIMPLIFY.

Solution

$$\frac{d}{dx}\left((-3x^2+5x-7)(3x-2)\right) = \left(\frac{d}{dx}(-3x^2+5x-7)\right)\cdot(3x-2) + (-3x^2+5x-7)\cdot\frac{d}{dx}(3x-2)$$

$$= (-6x+5)\cdot(3x-2) + (-3x^2+5x-7)(3)$$

Stop there

Example#2 Find derivative of $f(x) = (-3x^2+5x-7)\cdot e^{(x)}$ and simplify your answer.

STUDENTS SEEM TO FIND THIS REALLY HARD.

Solution

$$f'(x) = \left(\frac{d}{dx}(-3x^2+5x-7)\right)\cdot e^{(x)} + (-3x^2+5x-7)\cdot\left(\frac{d\,e^{(x)}}{dx}\right)$$

$$= (-6x+5)\cdot e^{(x)} + (-3x^2+5x-7)\cdot(e^{(x)})$$

remember $a \cdot c + b \cdot c = (a+b) \cdot c$

$$f'(x) = \left((-6x+5) + (-3x^2+5x-7)\right) \cdot e^{(x)}$$

$$= (-3x^2 - x - 2) \cdot e^{(x)}$$

Example #3 $\quad f(x) = 5x^7 \cdot \ln(x)$

(A) find $f'(x)$. (B) find $f'(1)$ (C) find $f'(e)$

Solution

$$f'(x) = \left(\frac{d}{dx} 5x^7\right) \cdot \left(\ln(x)\right) + \left(5x^7\right)\left(\frac{d}{dx} \ln(x)\right)$$

$$= (5 \cdot 7x^6) \cdot \ln(x) + 5x^7\left(\frac{1}{x}\right)$$

$$= 35x^6 \ln(x) + 5x^6$$

$$= 5x^6 \cdot (7\ln(x)) + 5x^6(1)$$

$$f'(x) = 5x^6(7\ln(x) + 1)$$

(B) $f'(1) = 5 \cdot 1^6 (7\ln(1) + 1)$

$= 5(7 \cdot 0 + 1)$

$= 5(1)$

$= 5$

remember

$e^0 = 1$

corresponding logarithm equation

$\ln(1) = 0$

(C) $f'(e) = 5 \cdot e^6 (7\ln(e) + 1)$

$= 5e^6(7(1) + 1)$

$= 5e^6(8)$

$= 40e^6$

remember $e^1 = e$

so $\ln(e) = 1$

The quotient rule — Used for finding the derivative of a quotient of functions $f(x) = \dfrac{top(x)}{bottom(x)}$

~~The obvious thing would be~~

$$\cancel{\frac{d}{dx}\left(\frac{top(x)}{bottom(x)}\right) = \frac{top'(x)}{bottom'(x)}}$$

~~The obvious thing is wrong.~~

The quotient rule is really ugly

$$\frac{d}{dx}\left(\frac{top(x)}{bottom(x)}\right) = \frac{top'(x) \cdot bottom(x) - bottom'(x) \cdot top(x)}{(bottom(x))^2}$$

Example $f(x) = \dfrac{2x}{3^{(x)}}$

Hard

(A) find $f'(x)$

(B) find equation of the line tangent to graph of f at $x = 3$.

Solution

(A) $f'(x) = \dfrac{\left(\frac{d}{dx}2x\right)\cdot 3^{(x)} - (2x)\left(\frac{d}{dx}3^{(x)}\right)}{\left(3^{(x)}\right)^2}$ quotient rule

$= \dfrac{(2)\cdot 3^{(x)} - (2x)\left(3^{(x)}\ln(3)\right)}{\left(3^{(x)}\right)^2}$ remember $\frac{d}{dx} b^{(x)} = b^{(x)}\ln(b)$

$= \dfrac{(2\cdot 3^x)(1) - (2\cdot 3^x)(x\ln(3))}{(3^x)^2}$ get ready to factor

$= \dfrac{(2\cdot 3^x)(1 - x\ln(3))}{(3^x)^2}$ factored out $2\cdot 3^x$

Can cancel 3^x

$$f'(x) = \frac{2\cdot(1 - x\ln(3))}{3^{(x)}}$$

(B) Tangent Line Problem

We need to build the equation

$$(y - f(a)) = f'(a)(x-a)$$

Get Parts

$a=3$

cancel one factor of 3

$$f(a) = f(3) = \frac{2\cdot(3)}{3^3} = \frac{2}{3^2} = \frac{2}{9}$$

$$f'(a) = f'(3) = \frac{2(1-(3)\ln(3))}{3^{(3)}}$$

Substitute Parts into equation

$$\left(y - \frac{2}{9}\right) = \frac{2(1-3\ln(3))}{3^3}(x-3)$$

Convert to slope intercept form

$$y - \frac{2}{9} = \left(\frac{2(1-3\ln(3))}{3^3}\right)x - \frac{2(1-3\ln(3))}{3^3}\cdot 3$$

$$y - \frac{2}{9} = (\sim\sim)x - \frac{2(1-3\ln(3))}{3^2}$$ cancel one factor of 3

$$= (\sim\sim)x - \frac{2(1-3\ln(3))}{9}$$

$$y - \frac{2}{9} = (\sim\sim)x - \frac{2}{9} + \frac{2(3)\ln(3))}{9}$$

$$y = \frac{2(1-3\ln 3)}{27}x + \frac{2\ln(3)}{3}$$

<u>example</u> $f(x) = \dfrac{e^x}{3x^2-5}$ find $f'(x)$

Solution

$$f'(x) = \frac{\left(\frac{d}{dx} e^x\right)(3x^2-5) - (e^x)\left(\frac{d}{dx} 3x^2-5\right)}{(3x^2-5)^2}$$

$$= \frac{(e^x)(3x^2-5) - (e^x)(6x)}{(3x^2-5)^2}$$

Note: cannot cancel $3x^2-5$

But we <u>can</u> factor out an e^x in numerator

$$= \frac{e^x\left((3x^2-5) - 6x\right)}{(3x^2-5)^2}$$

$$= \frac{e^x(3x^2-6x-5)}{(3x^2-5)^2}$$