Day 21 is Monday, March 11, 2013

Today continuing Section 5-1.

Goal: develop analytical tools that correlate behavior of graph of f and behavior of sign of f'.

Local Extrema

Definition of Local Max (or "Relative Max")

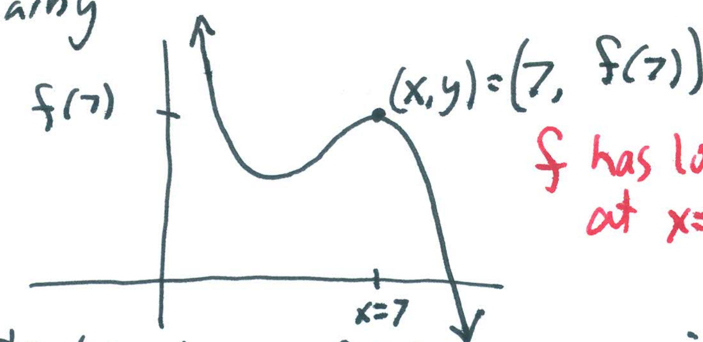
Words: f has a local max at $x = c$.

meaning: for all x values near c, but not equal to c,

$$f(c) > f(x).$$

graphical interpretation

On a graph of f, the point $(x,y) = (c, f(c))$ is the highest point nearby



f has local max at $x=7$

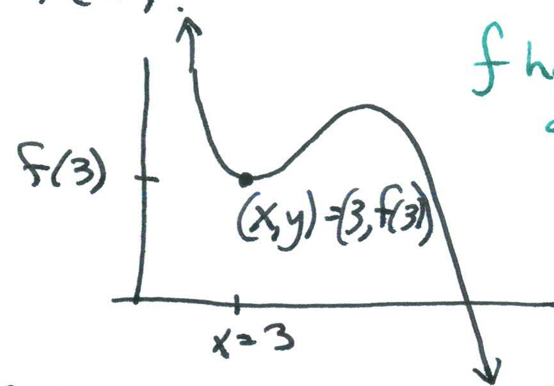
Notice: For the inequality to be true, $f(c)$ must exist.

Similar Definition for Local Min

words: f has local min at $x=c$

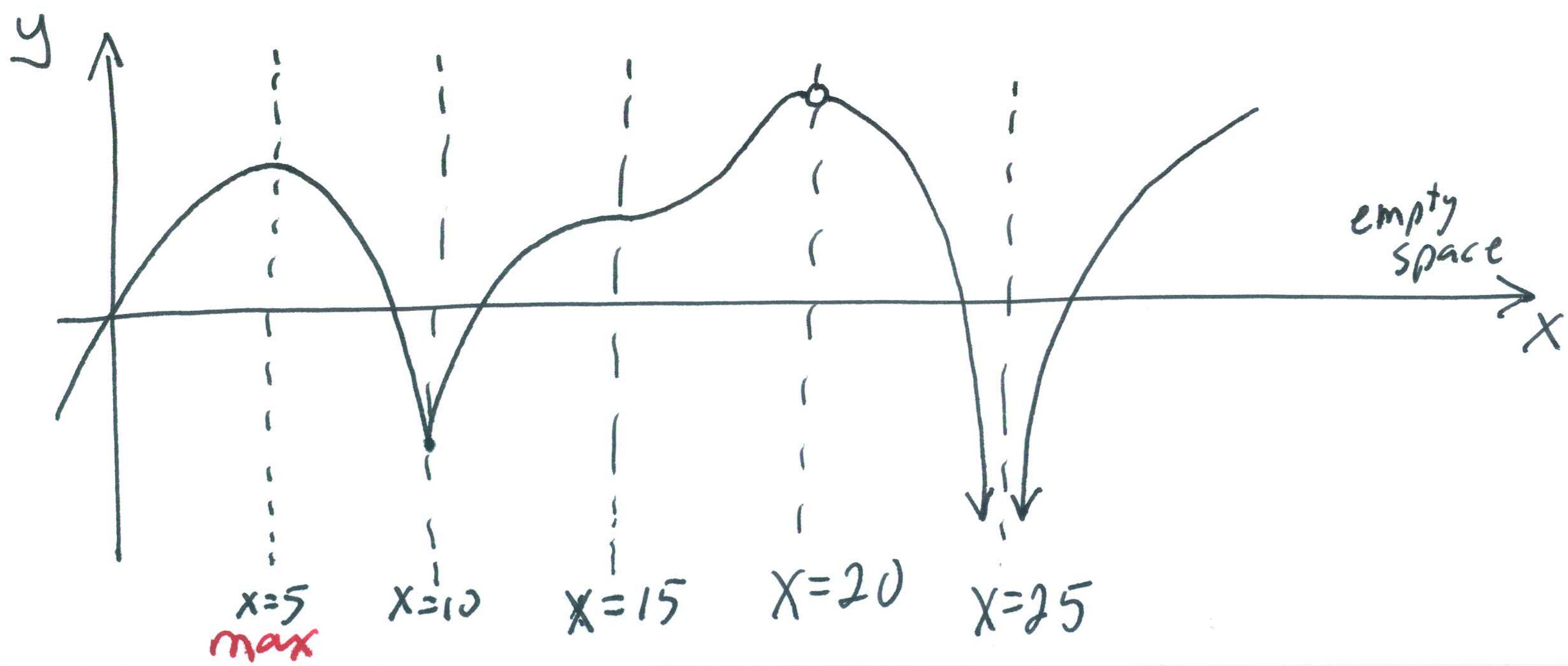
meaning: for all x values near c, but not equal to c,

$$f(c) < f(x)$$



f has local min at $x=3$

Sample graph for coming discussion



Question: what are the x-coordinates of all ~~tota~~ local maxs or local mins?

Local max at $x=5$

Not a local max at $x=20$ because graph has no y-value there.

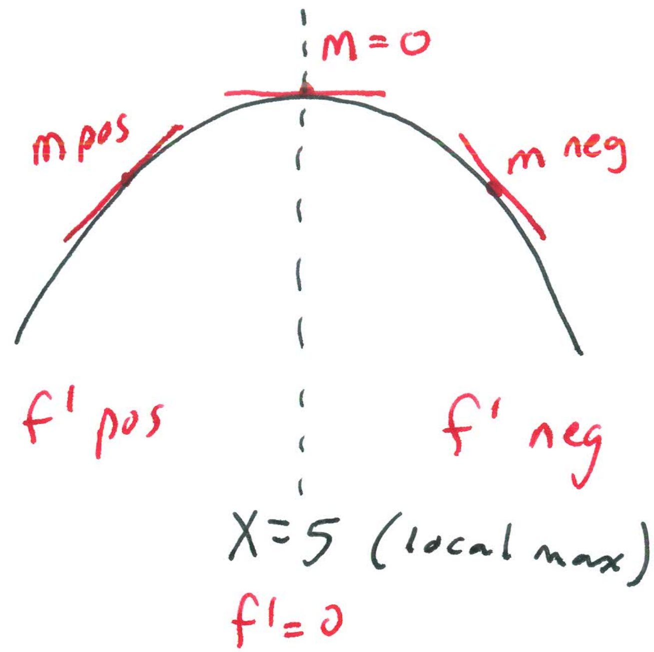
Local min at $x=10$

Not a local min at $x=25$ because no y-value.

Goal: Develop an analytic test that will determine where a function f has local max or min.

Consider behavior near $x = 5, 10, 15, 20, 25$

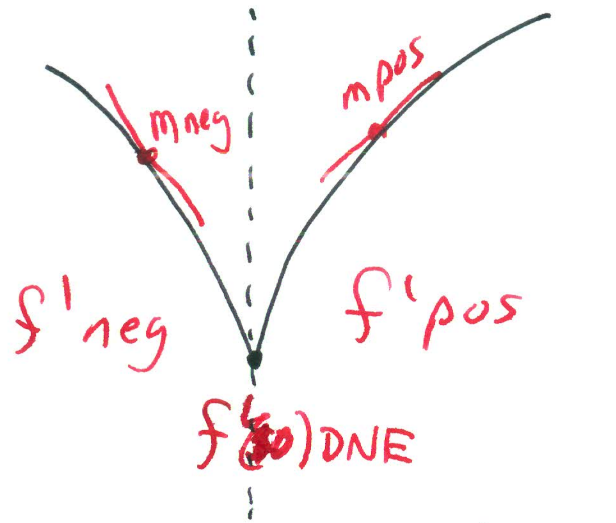
near $x=5$



at this local max,

- the y-value exists at $x=5$. $f(5)$ exists.
- $f'(5) = 0$ (horiz tangent at $x=5$)
- f' changes sign (from pos to neg) at $x=5$.

near $x=10$

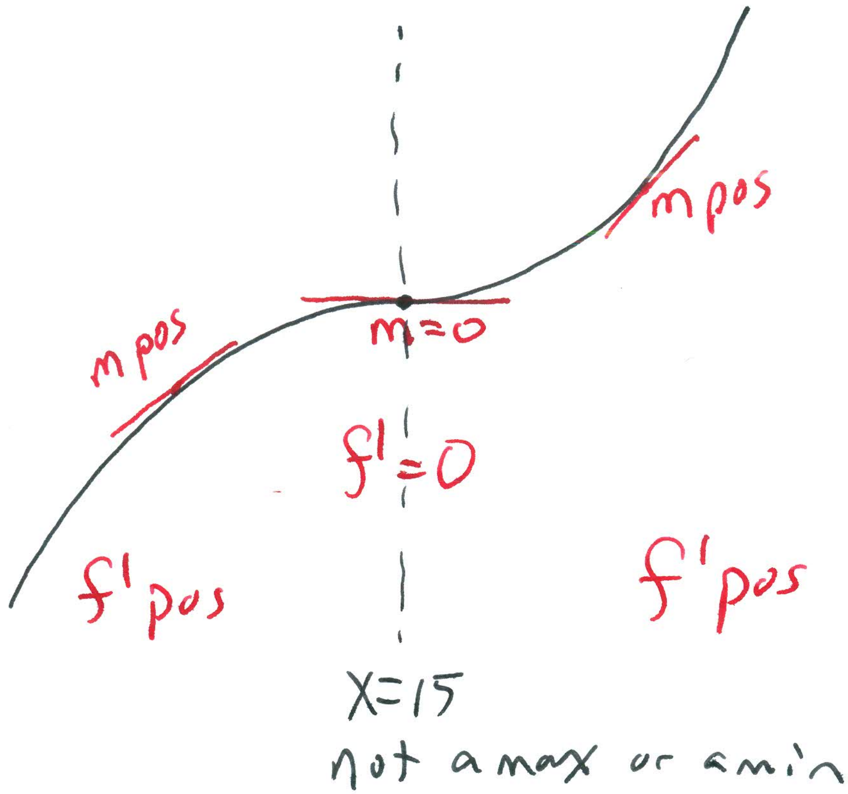


$x=10$ local min

at this local min

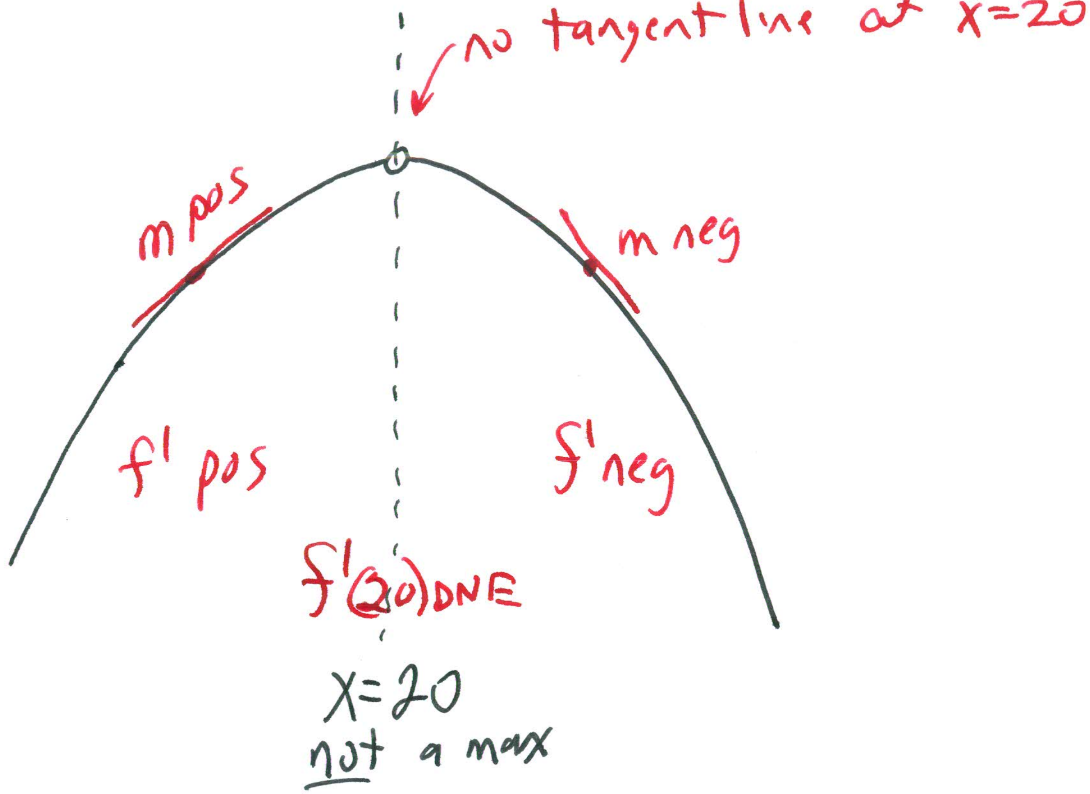
- The y-value exists at $x=10$. $f(10)$ exists
- $f'(10)$ DNE because no tangent line at $x=10$.
- f' changes sign (from neg to pos) at $x=10$.

near $x=15$



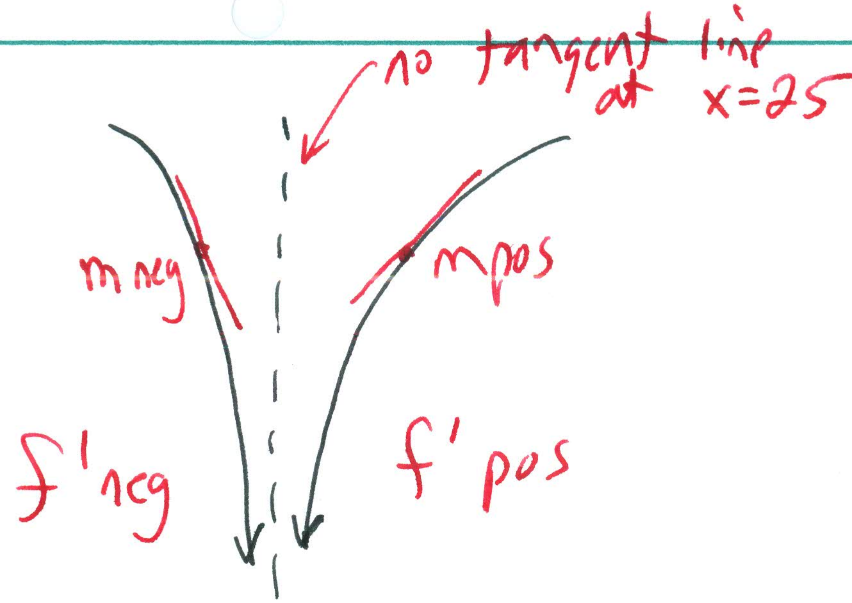
- The y-value exists at $x=15$. $f(15)$ exists.
- $f'(15)=0$ (horizontal tangent line at $x=15$)
- f' did not change sign at $x=15$. That's why there is no max or min at $x=15$.

near $x=20$



- the y value does not exist at $x=20$. $f(20)$ DNE.
- $f'(20)$ DNE because no tangent line
- f' changes sign at $x=20$. (from pos to neg)

near $x=25$



$x=25$
not a max or min

$f(25)$ DNE

$f'(25)$ DNE (because no tangent line at $x=25$

f' <u>does</u> change sign at $x=25$

Observation

At every $x=c$ where there is a local max or local min, the following thing occur:

- $f(c)$ exists
- $f'(c)=0$ or $f'(c)$ DNE
- f' changes sign at $x=c$.

Invent some useless but unfortunately common fancy notation.

* **Partition Number** A number $x=c$ is called a partition number for a function g if $g(c)=0$ or $g(c)$ DNE.

Rewrite our observation using terminology of partition numbers.

At every $x=c$ where there is a local max or local min, the following occur:

- $f(c)$ exists.
- $x=c$ is a partition number for f'
  (that is $f'(c)=0$ or $f'(c)$ DNE.
- f' changes sign at $x=c$.