Day 22 is Tuesday, March 12, 2013

The 1st Derivative Test for Local Extrema

If a function f passes all three of these tests at $x=c$

- $f'(c)=0$ or $f'(c)$ DNE } $x=c$ is partition number for f'
- $f(c)$ exists
- f' changes sign at $x=c$

} $x=c$ is a critical value for f

then the graph of f will have either a local max or a local min at $x=c$.

Fancy terminology

$f'(c)=0$ or $f'(c)$ DNE $\Longleftrightarrow$ "$x=c$ is a partition number for f'"

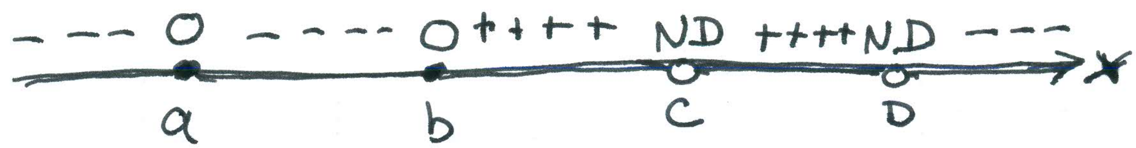
$f'(c)=0$ or $f'(c)$ DNE) and $f(c)$ exists } $\Longleftrightarrow$ "$x=c$ is a critical value for f"

Example 5-1 #10

sign chart for f' $\Rightarrow$ local extrema for f

A function $f(x)$ is continuous for all x-values, and has the following sign chart for f':

sign of f'



(A) Find partition numbers for f'

(B) Find critical values for f

(C) Find x-values where f has local max or min.

Solution

(A) $x = a, b, c, d$ because $f'(a) = 0$, $f'(b) = 0$, $f'(c)$ DNE, $f'(d)$ DNE.

(B) The fact that f is continuous everywhere tells us that $f(x)$ exists for all x-values. So $f(a)$, $f(b)$, $f(c)$, $f(d)$ all exist.

So all $x = a, b, c, d$ qualify to be called critical values for f.

(C) Local min at $x = b$ because f' changes ~~from~~ neg to pos.

Local max at $x = d$ because f' changes from pos to neg.

example $f(x) = -x^4 + 50x^2$

(A) find intervals where f is increasing or decreasing.

Present answer 3 ways:
- inequality notation
- interval notation
- number line

Solution

Strategy:
- find $f'(x)$
- make sign chart for $f'(x)$
- use sign chart to answer questions.

$$f(x) = -x^4 + 50x^2$$

$$f'(x) = -4x^3 + 100x$$

factor out -4

$$= -4(x^3 - 25x)$$

factor out x

$$= -4 \cdot x \cdot (x^2 - 25)$$

$$= -4 \cdot x \cdot (x+5)(x-5)$$

$$= -4(x+5)(x)(x-5)$$

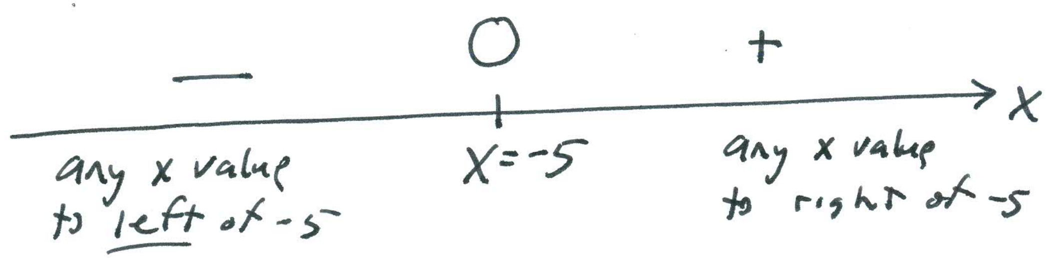
partition numbers: $x = -5, 0, 5$

(reordered the factors.)

Goal: Make sign chart for $f'(x)$.

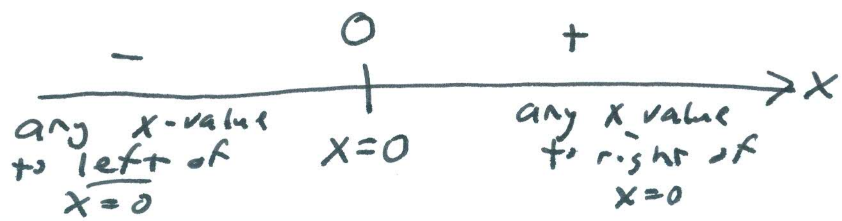
First, make sign chart for individual factors.

Sign chart for $(x+5)$



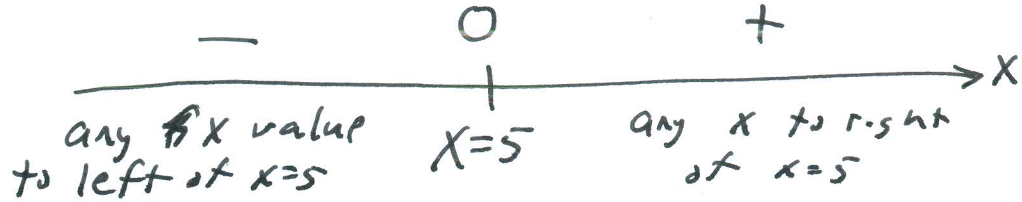
We see that the factor $(x+5)$ changes from neg to zero to pos at $x=-5$

Sign chart for (x)



So the factor (x) changes from $(-)$ to (0) to $(+)$ at $x=0$.

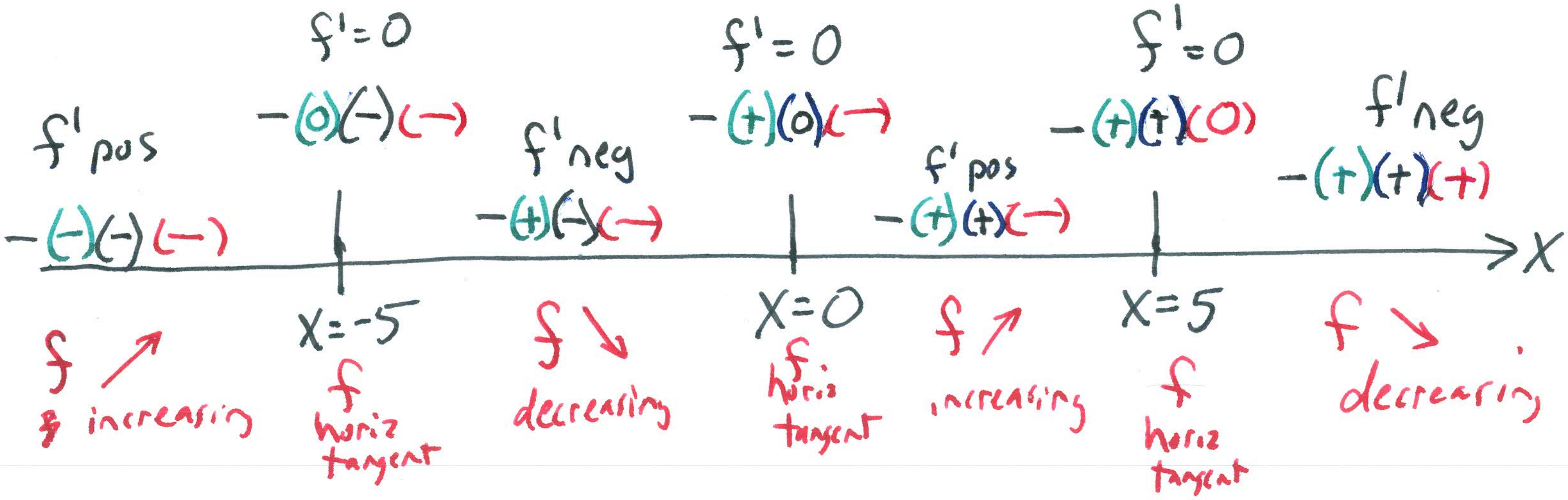
Similarly for the factor $(x-5)$:



The factor $(x-5)$ changes from $(-)$ to (0) to $(+)$ at $x=5$.

Combine these into a sign chart for $f'(x)$.

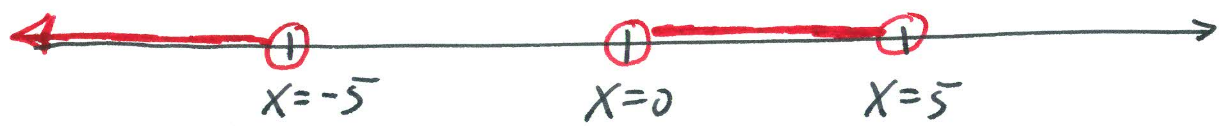
$$f'(x) = -4(x+5)(x)(x-5)$$



f is increasing for $x < -5$ and for $0 < x < 5$ because f' positive.
"inequality notation"

f is increasing on the intervals $(-\infty, -5)$ and $(0, 5)$ because f' pos.
(interval notation)

f is increasing on the following set:

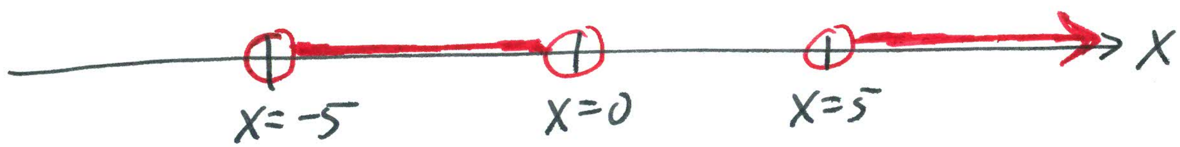


(picture involving number line)

f is decreasing for $-5 < x < 0$ and for $5 < x$ because f' neg
(inequality notation)

f is decreasing on the intervals $(-5, 0)$ and for $(5, \infty)$ because f' neg.

f is decreasing on the following set:



(B) Find x-coordinates of all local maxs.

Solution max at $x = -5$ and $x = 5$
because f' changes from pos to neg.

(C) Find x-coordinates of all local mins.

Solution $x = 0$ because f' changes from neg to pos.

(D) Find the y-coordinates of the extrema:

<u>Solution</u> substitute x-values into $f(x)$.

$$f(-5) = -(-5)^4 + 50(-5)^2$$

$$= -625 + 1250$$

$$= 625$$

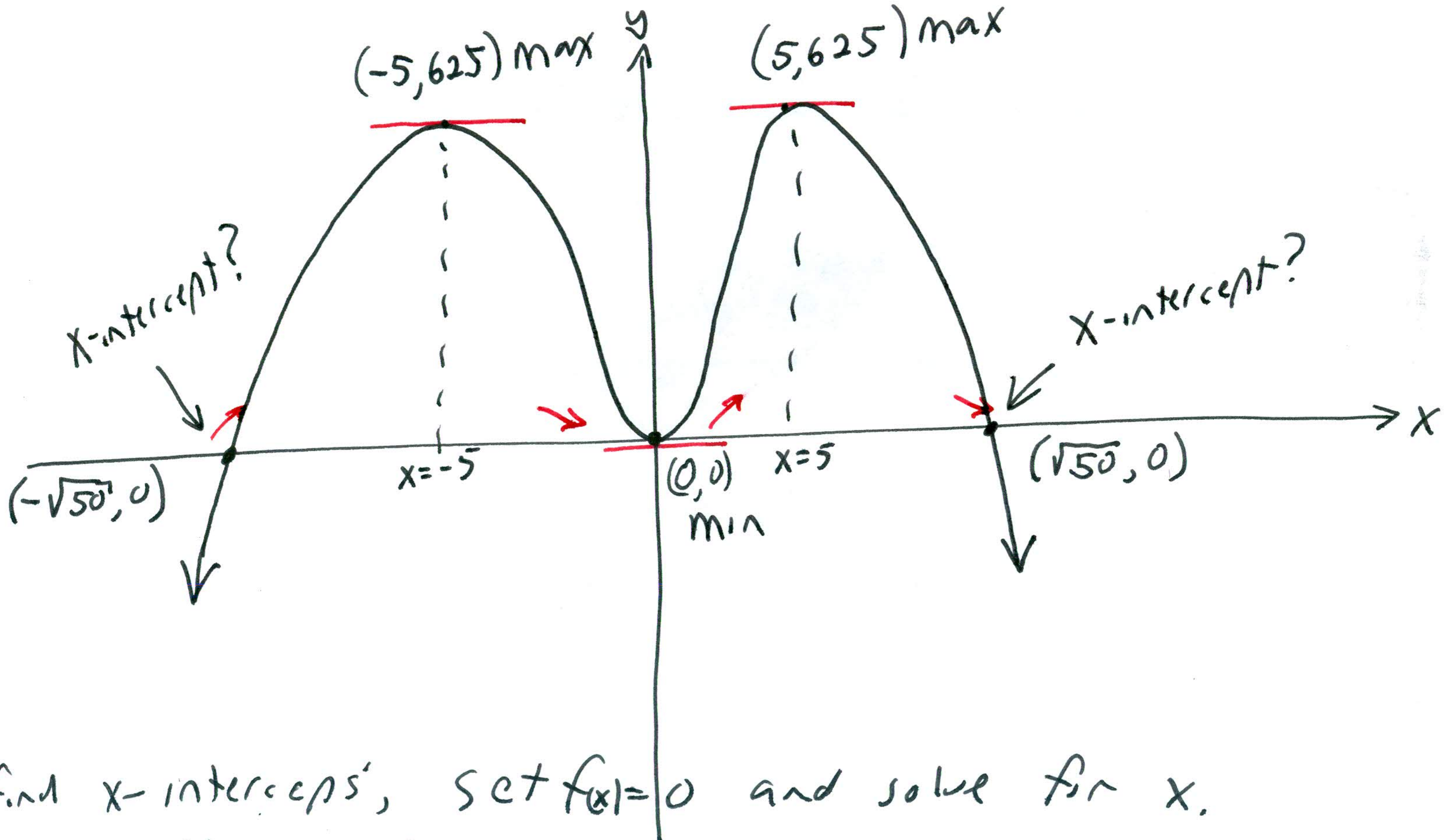
$$f(0) = -(0)^4 + 50(0)^2 = 0$$

$$f(5) = -(5)^4 + 50(5)^2$$

$$= -625 + 1250$$

$$= 625$$

(E) Sketch graph of $f(x)$, showing all horizontal tangent lines and putting (x,y) coordinates on all important points.



to find x-intercepts, set $f(x)=0$ and solve for x.

$0 = -x^4 + 50x^2$

$0 = -x^2(x^2 - 50)$

Solutions $x = 0$ and $x = +\sqrt{50}$ $x = -\sqrt{50}$

Start Section 5-2 2nd Derivatives & Concavity

The second derivative

Symbols: f'' , $\dfrac{d^2 f(x)}{dx^2}$

name: the second derivative of f.

meaning: the derivative of the derivative of f.

$$\frac{d^2 f(x)}{dx^2} = f''(x) = \frac{d}{dx}\left(f'(x)\right) = \frac{d}{dx}\left(\frac{d}{dx} f(x)\right)$$

Example

$$f(x) = -x^4 + 50x^2$$

Find $f''(x)$

Solution: $f'(x) = -4x^3 + 100x$

$$f''(x) = \frac{d}{dx}\left(-4x^3 + 100x\right)$$

$$= -4(3x^2) + 100(1)$$

$$f''(x) = -12x^2 + 100$$

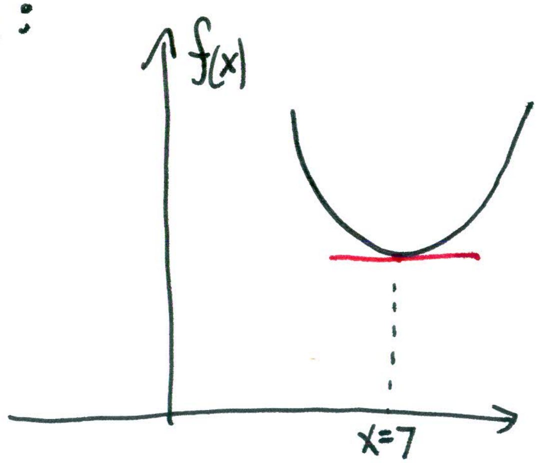
~~Relationship~~ Concavity

Definition of concave up at $x=c$

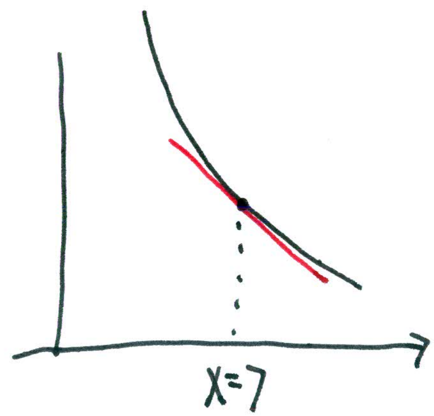
Words: "f is concave up at $x=c$"

Meaning: The graph of f has a tangent line at $x=c$ and for x values near c, the graph of f stays above the tangent line.

Picture:



f is concave up at $x=7$



f is concave up at $x=7$