Day 23 is Thursday, March 14, 2013

Continuing Section 5-2 2nd Derivatives + Concavity

Relationship between 2nd Derivative + Concavity

See Reference 6 on page 7 of course packet.

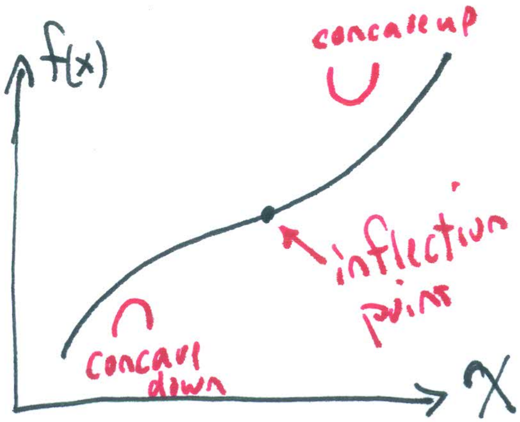
Today: Examples using Reference 6 Table to answer questions about behavior of the graph of a function.

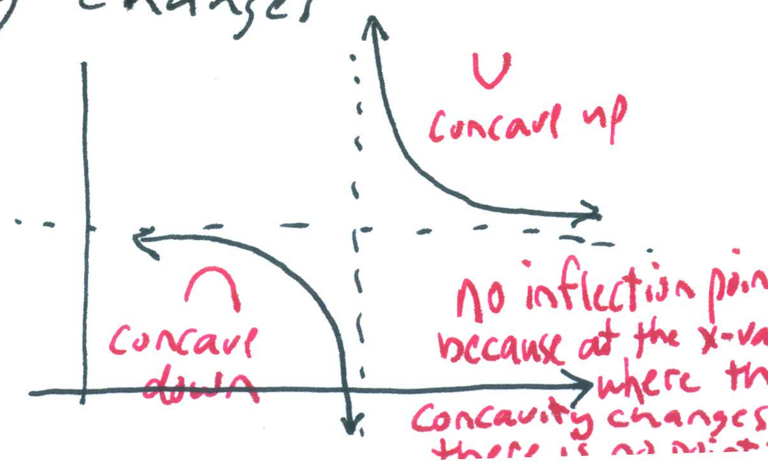
One more definition: Inflection Point

An inflection point is

- a point on a graph
- where the concavity changes

Example





<u>Examples</u> For the functions $f, g, h,$ do the following

(A) Find intervals where function is increasing or decreasing

(B) Identify x-coordinates of local max + mins.

(C) Find y-coordinates of local max + mins.

(d) Find intervals where function is concave up or down.

(e) Find x-coordinates of inflection points

(f) Find y-coordinates of inflection points.

First Example: $f(x) = -x^4 + 50x^2$

(A)
(B) these questions were answered on Tuesday.
(C) (see class notes pages 3-8)

(D) we need to analyze sign behavior of f''.

$$f(x) = -x^4 + 50x^2$$

$$f'(x) = -4x^3 + 100x$$

$$f''(x) = -12x^2 + 100$$

factor $f''(x)$ to get the partition numbers

Factor out -12

$$f''(x) = -12\left(x^2 - \frac{100}{12}\right)$$

$$= -12\left(x^2 - \frac{25}{3}\right) \quad \text{simplified}$$

$$= -12\left(x + \frac{5}{\sqrt{3}}\right)\left(x - \frac{5}{\sqrt{3}}\right) \text{ factored}$$

partition numbers: $x = -\frac{5}{\sqrt{3}}$, $x = +\frac{5}{\sqrt{3}}$
for f''

Sign chart for $f''(x)$.

Note: the factor $\left(x+\frac{5}{\sqrt{3}}\right)$ changes from $-$ to 0 to $+$
at $x = -\frac{5}{\sqrt{3}}$

the factor $\left(x - \frac{5}{\sqrt{3}}\right)$ changes from $-$ to 0 to $+$
at $x = +\frac{5}{\sqrt{3}}$

the term -12 in front is always <u>neg</u> !!

Put all this staff together to make sign chart for $f''(x)$

$$f''(x) = -12\left(x+\frac{5}{\sqrt{3}}\right)\left(x-\frac{5}{\sqrt{3}}\right)$$

f'' neg	$f''=0$	f'' pos	$f''=0$	f'' neg
$-(-)(-)$	$-(0)(-)$	$-(+)(-)$	$-(+)(0)$	$-(+)(+)$
	$x=-\frac{5}{\sqrt{3}}$		$x=\frac{5}{\sqrt{3}}$	
f concave down $\frown$	f inflection	f concave up $\smile$	f inflection	f concave down $\frown$

Conclude f ~~incre~~ concave up on the interval $\left(-\frac{5}{\sqrt{3}}, \frac{5}{\sqrt{3}}\right)$ because f'' is pos.

f concave down on the intervals $\left(-\infty, -\frac{5}{\sqrt{3}}\right)$ and $\left(\frac{5}{\sqrt{3}}, \infty\right)$ because f'' neg.

(E) f has inflection points at $x = -\frac{5}{\sqrt{3}}$ and $x = \frac{5}{\sqrt{3}}$.

(F) Find y-coordinates of inflection points

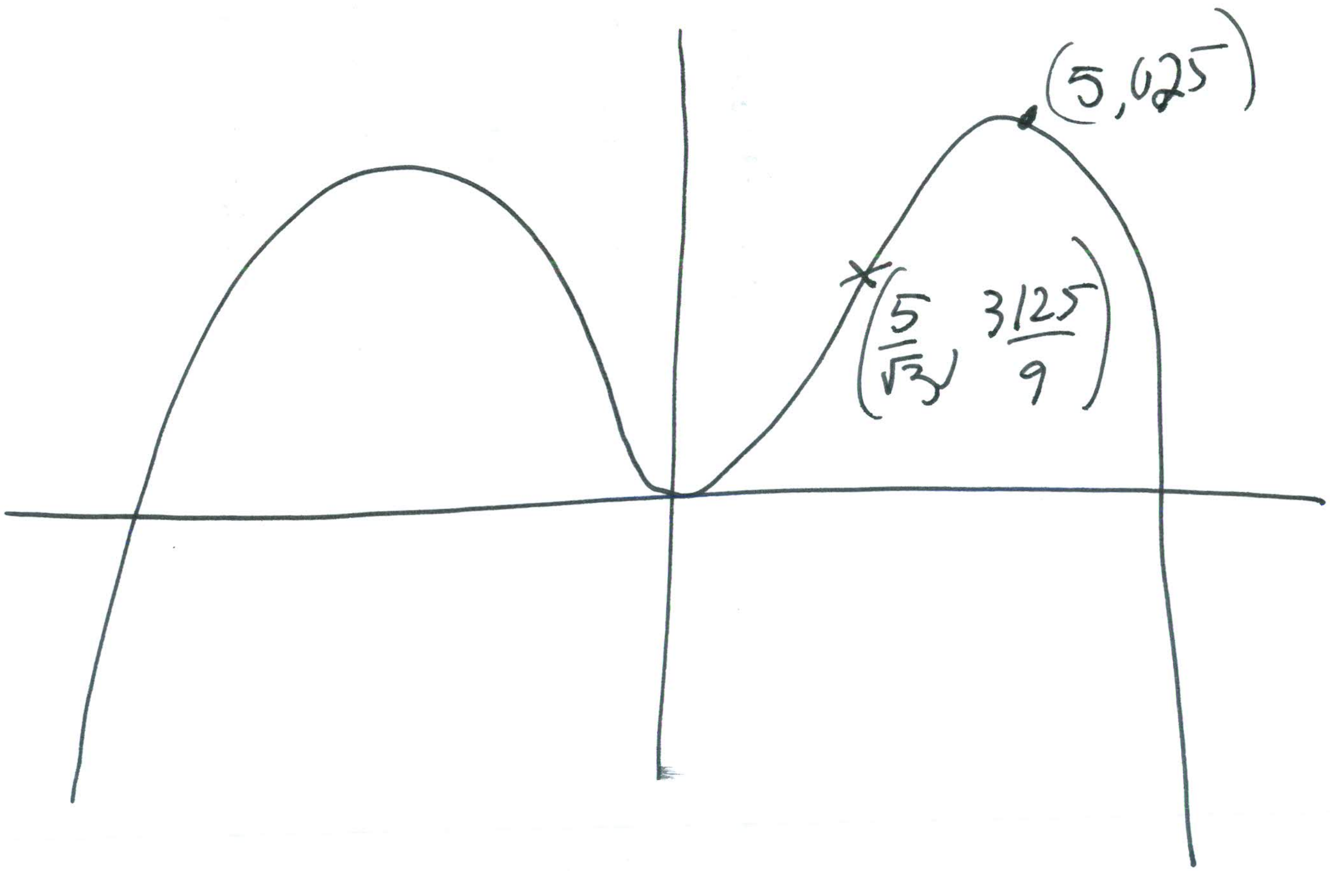
$$f\left(\frac{5}{\sqrt{3}}\right) = -\left(\frac{5}{\sqrt{3}}\right)^4 + 50\left(\frac{5}{\sqrt{3}}\right)^2$$

$$= -\left(\frac{25}{3}\right)^2 + 50\left(\frac{25}{3}\right)$$

$$= -\frac{625}{9} + \frac{1250}{3} = \frac{3125}{9}$$

So one inflection point is at $(x,y)=\left(\frac{5}{\sqrt{3}},\ \frac{3125}{9}\right)$,

the other one is at $(x,y)=\left(-\frac{5}{\sqrt{3}},\ \frac{3125}{9}\right)$



Example #2 Use function $g(x) = x e^{(-x)}$

Answer questions A, B, ..., F

(A) Strategy: Find ~~g~~ g'

Make sign chart for g'

analyze sign chart to answer the question.

$$g'(x) = \frac{d}{dx}\left(x e^{(-x)}\right)$$

product rule!!

$$= \left(\frac{d}{dx}x\right)\cdot e^{(-x)} + (x)\cdot\left(\frac{d}{dx}e^{(-x)}\right)$$

$$\frac{d}{dx}e^{cx} = ce^{cx}$$

$$= (1)\cdot e^{(-x)} + x\left((-1)e^{(-x)}\right)$$

now factor out e^{-x}

$$= (1-x)e^{(-x)}$$

$$= -(x-1)e^{(-x)}$$

Note the factor $(x-1)$ changes from neg to 0 to pos at $x=1$.

The factor $e^{(-x)}$ is always positive

because $e^{(\text{anything})} > 0$

So we can make a sign chart for

$$g'(x) = -(x-1)e^{(-x)}$$

g' pos

$-(-)+$

$g'=0$

$-(0)+$

g' neg

$-(+)+$

x

$x=1$

g ↗ increasing

g horiz tangent (max)

g ↘ decreasing