Day 25 is Tuesday, March 19, 2013

Recall Yesterday's Discussion of Section 5-5
Absolute Extrema

Definition: An absolute max for a function is a y-value that is the greatest y-value for all x-values that are in the domain.

An absolute min is a y-value that is the least y-value for all x-values in the Domain.

Theorem 2
The only places where absolute max/mins might occur is at x-values that are
- endpoints of the domain
- critical values.

(But absolute max/mins might not occur!)

<u>Theorem 1</u> If the domain of a function is a closed interval and the function is known to be continuous on that domain, then the function is guaranteed to have both an absolute max and an absolute min on that domain.

<u>The "Closed Interval Method"</u> Used for finding the absolute max & absolute min for a function f that is <u>known</u> to be continuous on a closed interval.

- Find the critical values of f
- List the important x-values (the x-values where there could possibly be an absolute max or min)
  - the critical values in the domain
  - the endpoints
- Find the corresponding y-values.
- Identify the greatest and least y-values on that list. These will be the absolute max and absolute min.

Example Let $f(x) = x^4 - 6x^2 + 5$

Find absolute extrema, if they exist, on the interval $[-3, 2]$.

Solution

Observe: our domain $[-3,2]$ is a closed interval.

and the function f is continuous.

So Theorem I tells us that there will be both an absolute max & absolute min, and we know that we should use the "closed interval method".

Start by finding the critical values of f.

Set $f'(x) = 0$ and solve for x. This will get us the x-values that are the partition numbers for f!

But these will automatically also be critical values for f because we know $f(x)$ exists everywhere. (f is continuous)

$f(x) = x^4 - 6x^2 + 5$

$f'(x) = 4x^3 - 12x + 0$

$= 4x^3 - 12x$ now factor this

$= 4x(x^2 - 3)$

Check

$4x(x^2 - 3) = 4x^3 - 12x$ ✓

factor some more

$= 4x(x + \sqrt{3})(x - \sqrt{3})$

Check

$(x+\sqrt{3})(x-\sqrt{3}) = x^2 + \sqrt{3}x - x\sqrt{3} - \sqrt{3}\sqrt{3}$

$= x^2 - 3$ ✓

partition numbers for f'

$x = 0$

$x = -\sqrt{3} \approx -1.732$

$x = \sqrt{3} \approx 1.732$

These are also the critical values for f.

List important x-values (in order)

important x-values	corresponding y-values
$x=-3$ (endpoint)	$f(-3)=32$ ← absolute max is $y=32$
$x=-\sqrt{3}$ (critical)	$f(-\sqrt{3})=-4$
$x=0$ (critical)	$f(0)=5$
$x=\sqrt{3}$ (critical)	$f(\sqrt{3})=-4$
$x=2$ endpoint	$f(2)=-3$

absolute min is $y=-4$

$f(-3)=(-3)^4-6(-3)^2+5 = 81-6(9)+5 = +81-54+5=32$

$f(-\sqrt{3})=(-\sqrt{3})^4-6(-\sqrt{3})^2+5 = 9-6(3)+5 = 9-18+5=-4$

$f(0)=5$

$f(\sqrt{3})=-4$

$f(2)=(2)^4-6(2)^2+5 = 16-6(4)+5=16-24+5=-3$

Conclude: Absolute max is $y = 32$ (occurs at $x = -3$)

Absolute min is $y = -4$ (occurs at $x = -\sqrt{3}$ and $x = \sqrt{3}$)

Important Remark:

What if you don't bother finding the critical values, and you just check all the x-values?

x	y-values
−3	
−2	
−1	
0	
1	
2	

This is no good! It is not "all" the x-values, It is only the integers. It is not a valid way to find absolute max + mins.

Another Example Same function $f(x) = x^4 - 6x^2 + 5$

Find absolute max & min on interval $[-1, 2]$

Solution

we know that the critical values are

~~$x = -\sqrt{3} \approx -1.7$~~ not in our interval

$x = 0$

$x = \sqrt{3} \approx 1.7$

List important x-values	
$x = -1$ (endpoint)	$f(-1) = (-1)^4 - 6(-1)^2 + 5 = 1 - 6(1) + 5 = 0$
$x = 0$ (critical)	$f(0) = 5$ ← abs max is $y = 5$
$x = \sqrt{3}$ (critical)	$f(\sqrt{3}) = -4$ ← abs min is $y = -4$
$x = 2$ (endpoint)	$f(2) = -3$

Conclusion: Abs max is $y = 5$ (occurs at $x = 0$)
Abs min is $y = -4$ (occurs at $x = \sqrt{3}$)

<u>Remark</u> Changing the domain affects the values of the max & mins.

<u>Question</u>: What if the interval is <u>not</u> closed, or what if the function f is <u>not</u> continuous?

Will we even have absolute max & min?

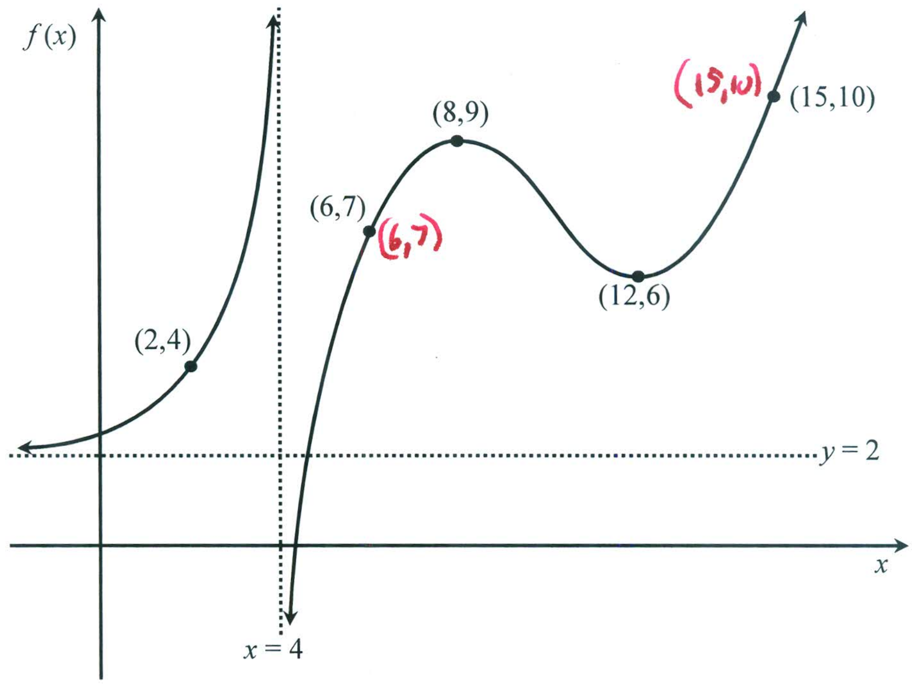
<u>Class Drill 14</u> on page 40.

Answer: ~~the~~ Might have Abs Max or Min, but not necessarily.

Class Drill 14: Relative and Absolute Extrema

The *Extreme Value Theorem* says that if a function f is continuous on a closed interval $[a,b]$, then f will have both an absolute maximum and an absolute minimum on that interval. In this drill, you investigate what can happen when f is not continuous or the interval is not closed.

The graph of a function f is shown at right. Fill in the table below.



Interval	Relative Maxima in that interval	Relative Minima in that interval	Absolute Max in that interval	Absolute Min in that interval
[6,15]	y=9 y=10	y=6 y=7	y=10	y=6
(6,15)	y=9	y=6	none	y=6
(8,15)	none	y=6	none	y=6
[2,12]	y=9	y=4 y=6	none	none
(2,12)				
(4,∞)				

Example of finding Abs Max + Abs Min for a function ~~th~~ on interval that is **not** closed.

Example $f(x) = x^4 - 6x^2 + 5$

Find all absolute extrema on the interval $(-\infty, \infty)$.

Solution

Observe: f is continuous, but interval is not closed. So we are not guaranteed absolute max or min.

Will we even have abs max or min??

Observe f is even degree polynomial, with positive leading coefficient, so both ends go up.

Graph will look like this

Will be no absolute max, but there <u>will</u> <u>be</u> an absolute min. (the lowest bump)

Where should we look for the abs min?

Theorem 1 says abs min can only occur at endpoints or at critical values.

We don't have any endpoints. So the abs min will occur at ~~at~~ a critical value.

Earlier today (page 5) we found these critical values and their y-values

$x = -\sqrt{3}$	$y = -4$
$x = 0$	$y = 5$
$x = \sqrt{3}$	$y = -4$

So absolute min is $y = -4$, occurs at $x = \sqrt{3}$ and $x = -\sqrt{3}$.

function has no absolute max

We will start with this example on thursday:

$$f(x) = 20 - 4x - \frac{250}{x^2}$$

Find all absolute extrema on the interval $(0, \infty)$.