Day 26 is Thursday, March 21, 2013

Final example on Section 5-5 material

$$f(x) = 20 - 4x - \frac{250}{x^2}$$

Find all absolute extrema on the interval $(0, \infty)$

Solution

Notice: the interval $(0, \infty)$ is not closed, So we are not guaranteed an abs. max or an abs min.

Observe also that $x=0$ is not in the domain because $\frac{250}{0^2}$ DNE. So $f(x)$ is not continuous at $x=0$. But $f(x)$ is continuous everywhere else. In particular, f is continuous on our interval $(0, \infty)$

So we don't know if f will even have max or min, and we don't know anything about the shape of its graph.

But we <u>do</u> know that if there <u>are</u> any max/mins, they will have to occur at critical values of f.

So let's start by finding the critical values of f.

~~Look for~~ First find <u>partition</u> numbers for f'.

(the x-values that cause $f'(x) = 0$ or $f'(x)$ DNE)

Must find $f'(x)$

$$f(x) = 20 - 4x - \frac{250}{x^2}$$

Rewrite f(x) to make derivative easier

$$f(x) = 20 - 4x - 250x^{-2}$$

$$f'(x) = 0 - 4 - 250(-2x^{-3})$$
$$= -4 + 500x^{-3}$$
$$f'(x) = -4 + \frac{500}{x^3}$$

Notice $x=0$ causes $f'(0)$ DNE, so $x=0$ is a partition number for $f'(x)$.

Look for x values that cause $f'(x)=0$

$$0 = f'(x)$$
$$= -4 + \frac{500}{x^3}$$

add 4 to both sides

$$4 = \frac{500}{x^3}$$

Multiply both sides by x^3

$$4X^3 = 500$$

Divide by 4

$$X^3 = \frac{500}{4}$$

$$X^3 = 125$$

$$X = 5$$

Partition numbers for $f'(x)$ are $x=0$, $x=5$.

But only $X=5$ is qualified to be called a ~~partition numbe~~ critical value for f, because $f(5)$ exists while $f(0)$ DNE.

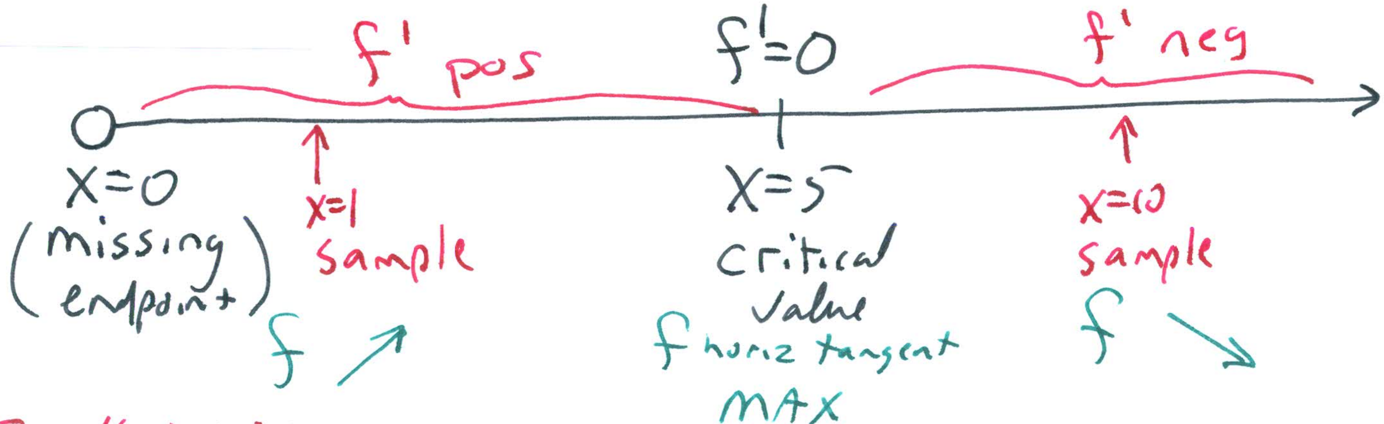
So $X=5$ is our only critical value.

Now we need to figure out if $x=5$ is a max or a min or neither.

Two ways of doing this:

<u>Method #1</u> Investigate the <u>sign</u> of f'.

Sign chart for $f'(x)$ on the interval $(0,\infty)$



$$f'(1) = -4 + \frac{500}{(1)^3} = -4 + \frac{500}{1} = -4 + 500 = \text{positive}$$

$$f'(10) = -4 + \frac{500}{10^3} = -4 + \frac{500}{1000} = -4 + \frac{1}{2} = \text{negative}$$

Since

0 ———————— $x=5$ ————————

f increasing | f horiz tangent | f decreasing

we know that $x=5$ is the location of the absolute max on the interval $(0,\infty)$

The value of the abs max is the y-value

$$f(5) = 20 - 4(5) - \frac{250}{(5)^2} = 20 - 20 - \frac{250}{25}$$

$$= -10$$

absolute max is $y = -10$
(it occurs at $x=5$)

End of method #1

Method #2 for analyzing the critical value $x=5$

Investigate concavity behavior of f

by studying sign of f''.

$$f(x) = 20 - 4x - \frac{250}{x^2} = 20 - 4x - 250x^{-2}$$

$$f'(x) = -4 + \frac{500}{x^3} = -4 + 500x^{-3}$$

$$f''(x) = 0 + 500(-3x^{-4}) = -1500x^{-4}$$

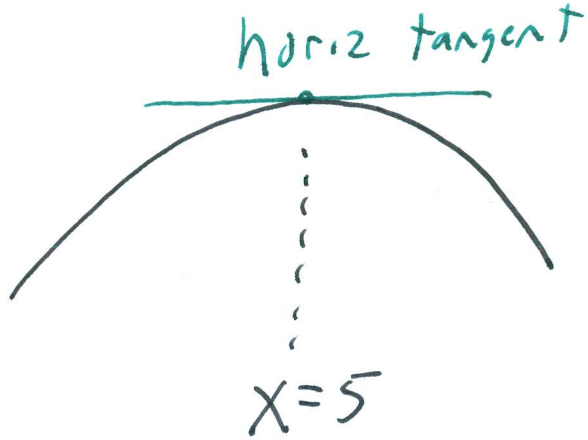
$$f''(x) = -\frac{1500}{x^4}$$

for $x>0$, x^4 will be positive

so $\frac{1500}{x^4}$ will be positive

so $-\frac{1500}{x^4}$ will be negative.

Since $f''(x)$ is negative for all $x>0$,
this tells us us that f is concave down
on the whole interval $(0, \infty)$



So the $x=5$ is the location of the absolute max
on the interval $(0,\infty)$
~~So~~ The value of the abs max is $y=f(5)=-10$

end of method #2

Remark: Method #2 is called the "Second Derivative Test"

Section 5-6 Optimization (Applied max/min problems)

Possible complications

- may be presented as word problems
- may have domains that are not closed intervals
- you ~~may~~ probably have to figure out the domain
- may involve more than one variable.

Example

A company makes & sells x cameras per week. The price demand equation is $p = 300 - \frac{x}{30}$

The cost function is $C(x) = 90{,}000 + 30x$.

(A) If the goal is to maximize weekly Revenue, what price should the company charge for each camera, and how many cameras should be made each week?

Solution

Must find max for the Revenue function

$$R(x) = \text{Demand} \cdot \text{price} = x \cdot p$$

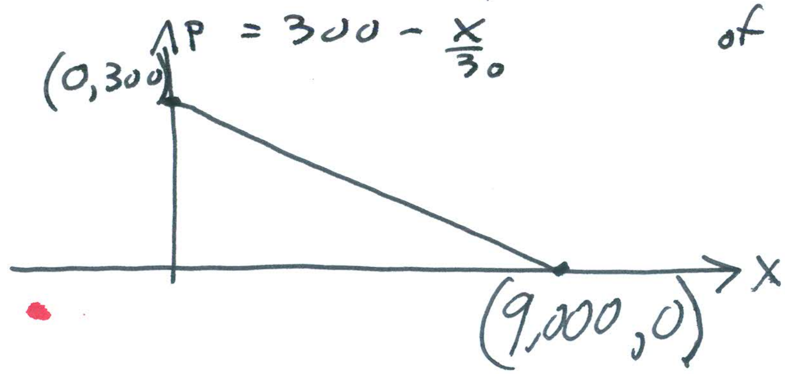
$$= x \cdot \left(300 - \frac{x}{30}\right)$$

$$= 300x - \frac{x^2}{30}$$

What is the domain?

Must have $x \geq 0$.

But also must have price $p \geq 0$. Consider graph of price



of the form $y = mx + b$

y-intercept $b = 300$

slope $m = \left(-\frac{1}{30}\right)$

Set $p = 0$, solve for x

$$0 = 300 - \frac{x}{30}$$

$$\frac{x}{30} = 300$$

$$x = 9{,}000$$

must have $0 \leq x \leq 9{,}000$

So the domain is $[0, 9{,}000]$

So we have a continuous function $R(x) = 300x - \frac{x^2}{30}$ defined on a closed interval $[0, 9000]$

So there <u>will</u> be an abs max & abs min.

They must occur at endpoints + critical values.
(the closed interval method)

Find the critical values

$$R(x) = 300x - \frac{x^2}{30} = 300x - \left(\frac{1}{30}\right)x^2$$

$$R'(x) = 300(1) - \left(\frac{1}{30}\right)(2x) = 300 - \frac{x}{15}$$

Set $R'(x) = 0$ and solve for x.

$$0 = 300 - \frac{x}{15}$$

add $\frac{x}{15}$ to both sides

$$\frac{x}{15} = 300$$

Multiply both sides by 15

$$x = 300(15) = 4500 = \text{critical value.}$$

List of important x-values

x	corresponding $R(x)$ values
$x=0$ endpoint	$R(0) = 300(0) - \frac{0^2}{30} = 0$
$x=4500$ critical	$R(4500) = 300(4500) - \frac{(4500)^2}{30} = 675{,}000$
$x=9000$ endpoint.	$R(9000) = 0$

Conclude max revenue is 675,000 dollars per week.

The company should make $x=4500$ cameras per week.

The selling price is obtained by substituting $x = 4500$ into the price equation.

$$p = 300 - \frac{x}{30}$$

$$p = 300 - \frac{4500}{30}$$

$$= 300 - 150$$

$$p = 150$$

The company should sell the cameras for \$150 each.