Day 27 is Monday, March 25, 2013

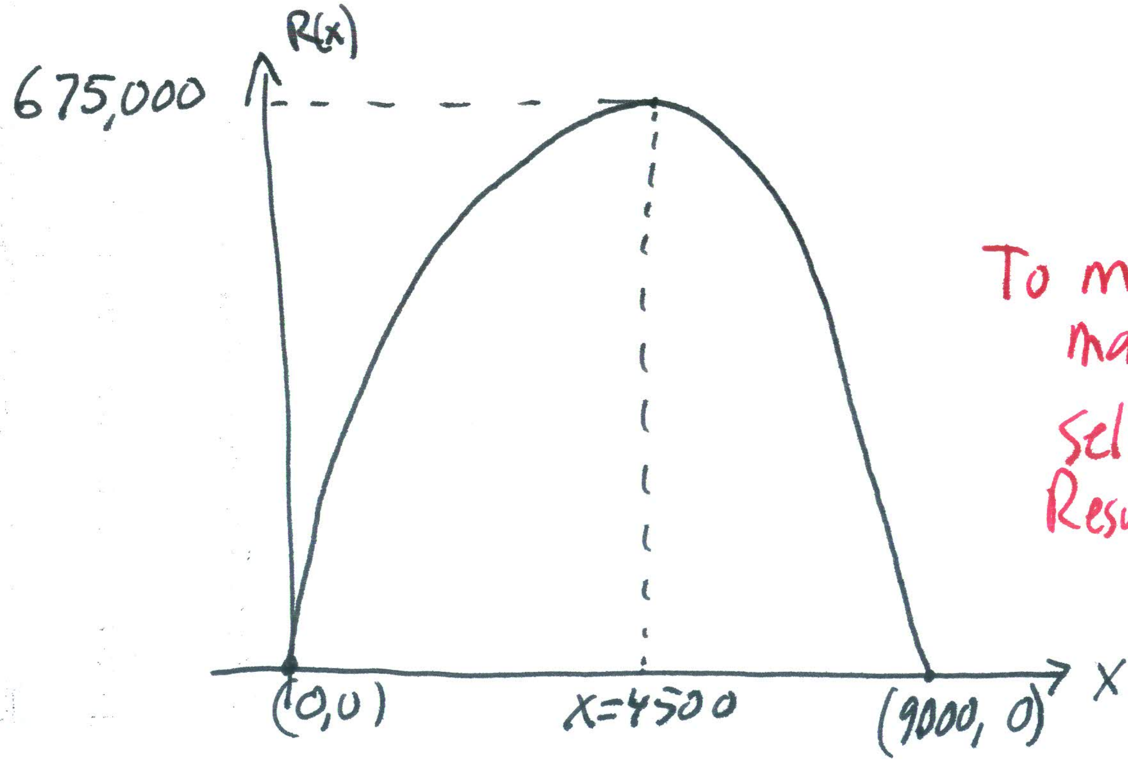
Reviewing + Continuing Thursday's example involving company making cameras.

Plot the Revenue function. $R(x) = 300x - \frac{x^2}{30}$

Will be parabola facing down.

Three known points (from Thursday)

x	$R(x)$
0	0
4500	675,000
9000	0



To maximize weekly Revenue, make 4500 cameras per week

Sell them for \$150 per camera

Resulting Revenue will be \$675,000 per week.

(B) <u>New Question</u>: If the goal is to maximize the weekly <u>Profit</u>, how many cameras should be made each week, and what should be the selling price per camera?

<u>Solution</u>

Profit = Revenue − Cost

$$P(x) = R(x) - C(x)$$

(capital P)

$$= \left(300x - \frac{x^2}{30}\right) - (\underbrace{90{,}000}_{\text{"fixed costs"}} + \underbrace{30x}_{\text{"variable costs"}})$$

$$= -\frac{x^2}{30} + 270x - 90{,}000$$

domain is still $0 \le x \le 9000$ because $P(x)$ equation used $R(x)$, and the $R(x)$ equation is only valid on that domain

Goal: maximize the function

$$P(x) = \frac{-x^2}{30} + 270x - 90{,}000$$

on the domain $0 \le x \le 9000$

Find critical value by setting $P'(x) = 0$ and solving for x.

$$P'(x) = \left(-\frac{1}{30}\right)(2x) + 270$$

$$= -\frac{x}{15} + 270$$

Set $P'(x) = 0$:

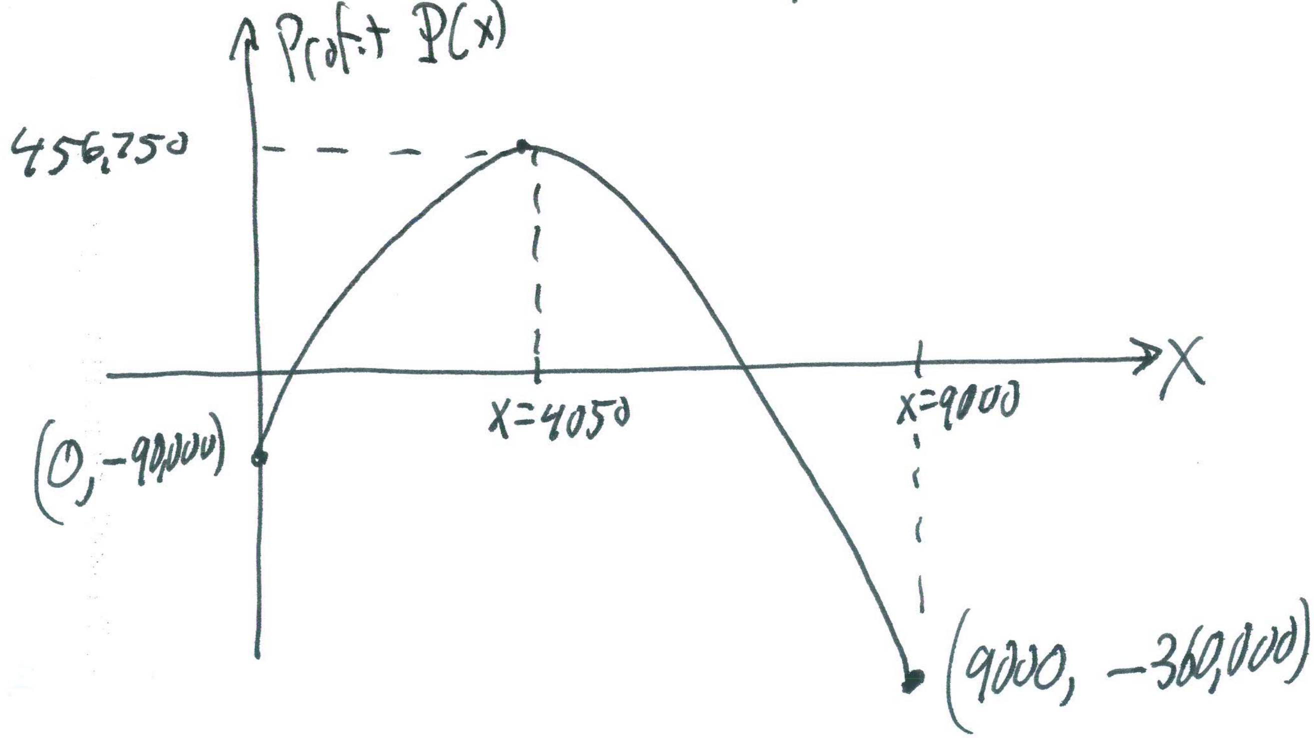
$$0 = -\frac{x}{15} + 270$$

$$\frac{x}{15} = 270$$

$$x = 4050$$

~~F~~ → List of important x-values	Values of $P(x)$	
$x=0$ endpoint	$-90{,}000$	(operating at a loss!)
$x=4050$ critical	$456{,}750$	
$x=9000$ endpoint	$-360{,}000$	(big loss)

Quick graph of the profit function



The selling price per camera should be

small $\rightarrow p = 300 - \frac{x}{30}$ use $x = 4050$

result is

$p = 300 - \frac{4050}{30} = \$165$ per camera.

To maximize weekly profit, we should charge more for the cameras

Charge \$165 (instead of ~~\$150~~ \$150)

We will sell fewer camera per week

(4050 per week instead of 4500 per week)

Revenue will go down, but so will costs,

Profit will be maximized.

Another optimization Example (Similar to exercise 5-6 #1)

Find two positive numbers x, y such that

- the product $x \cdot y = 9000$
- the sum $10x + 25y$ is minimized.

Notice complications

Two variables

No clear given function, no clear domain.

Solution

Step 1 Write equation I involving x & y

$$x \cdot y = 9000$$

Step 2 Write equation II involving x & y and the letter S for "sum".

$$S = 10x + 25y$$

Step 3 Solve Equation I for y in terms of x.
Call that the New equation I

Equation I $xy = 9000$
divide by x

$$y = \frac{9000}{x} \quad \text{New Equation I}$$

Step 4 Substitute Equation I into equation II and simplify

$$S = 10x + 25y$$

$$= 10x + 25\left(\frac{9000}{x}\right)$$

$$= 10x + \frac{25(9000)}{x}$$

Observe that this gives S as a function of x.
Rewrite with function notation

$$S(x) = 10x + \frac{25(9000)}{x}$$

Our goal is to find the value of x that minimizes $\$(x) = 10x + \frac{25(9000)}{x}$

on the domain $x > 0$

that is, the interval $(0, \infty)$

(This is similar to example from start of class on Thursday March 21)

Find critical values of $\$(x)$

$$\$(x) = 10x + 25(9000)(x^{-1})$$

$$\$'(x) = 10 + 25(9000)(-1)x^{-2})$$

$$= 10 - \frac{25(9000)}{x^2}$$

Set $\$'(x) = 0$ and solve for x

$$0 = 10 - \frac{25(9000)}{x^2}$$

add $\frac{25(9000)}{x^2}$ to both sides

$$\frac{25(9000)}{x^2} = 10$$

Multiply both sides by x^2

$$25(9000) = 10x^2$$

Divide both sides by 10

$$\cancel{250}\ 25(900) = x^2$$

$$x = \pm\sqrt{25(900)} = \pm\sqrt{25}\sqrt{900} = \pm(5)(30)$$

$$x = \pm 150$$

not in domain

Critical values are $x = -150$, $x = 150$

We need to see if $S(x)$ has a max or a min at $x=150$.

Investigate $S''(x)$ to determine concavity

$$S'(x) = 10 - 25(9000)x^{-2}$$

$$S''(x) = 0 - 25(9000)(-2x^{-3})$$

$$= \frac{25(9000)(2)}{x^3}$$

x is pos

so x^3 is pos

so $\frac{25(9000)(2)}{x^3}$ is pos

~~$S''(150)$ will be po~~

Observe S'' is pos for all $x>0$
So S is concave up.
So $x=150$ is an absolute min

on the interval ~~[illegible]~~ $(0,\infty)$

So use $x=150$

and $y = \frac{9000}{x} = \frac{9000}{150} = 60$

use $x=150$, $y=60$