Day 33 is Monday, April 8, 2013

Returning to example from end of class on thursday.

example:

Find the particular antiderivative

$$S(t) = \int 20 - 20e^{(-.05t)}\,dt$$

that satisfies $S(0) = 0$

Result: General Antiderivative:

$$S(t) = 20t + 400e^{(-.05t)} + K$$

undetermined constant

The particular Antiderivative that satisfies the additional condition $S(0)=0$ is

$$S(t) = 20t + 400e^{(-.05t)} - 400.$$

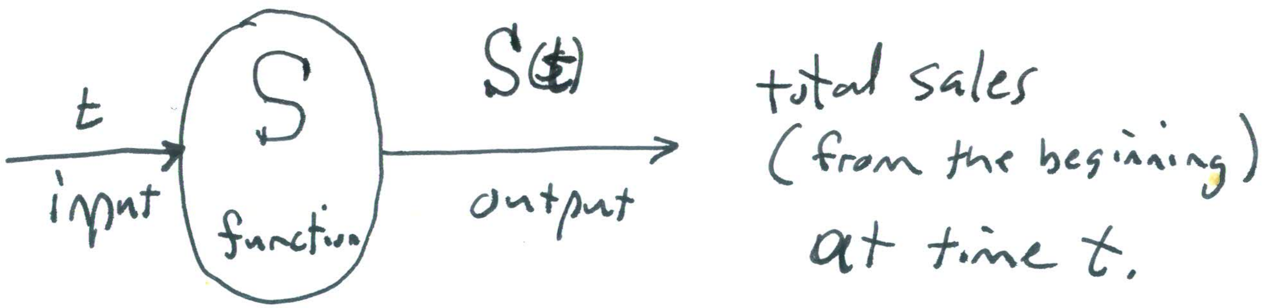
New example with the same underlying math, but presented in a different way.

Car company is selling new model of car.

t = variable representing time (in months) since the car was introduced.

$S(t)$ = <u>function</u> representing total sales (in millions of dollars) at ~~the end of the~~ time t.

capital



We are told that $S'(t) = 20 - 20e^{-.05t}$

Note: we are given the derivative of $S(t)$. We are not given $S(t)$

Questions:

(A) Find the formula for $S(t)$

(B) Use that formula to estimate the total sales at the end of the 1st year.

Solution

Observation: We know that $S(0) = 0$ because the total sales at time 0 will necessarily be 0.

So we are looking for a function $S(t)$ that ~~Satisf~~ Satisfies these two conditions:

- $S'(t) = 20 - 20e^{(-.05t)}$
- $S(0) = 0$

Solution, continued

We see that the two conditions are equivalent to the two conditions from Thursday's example (see page 1)

- $S(t) = \int 20 - 20e^{(-.05t)} dt$
- $S(0) = 0$

So the function $S(t)$ that we need will just be the one from Thursday:

$$S(t) = 20t + 400e^{(-.05t)} - 400$$

Answer to part (A)

(B) To find total sales at end of 1st year, just plug in $t = 12$.

$$S(12) = 20(12) + 400e^{(-.05)(12)} - 400$$

$$\approx 59.525$$

Conclude that the total sales at the end of 12 months is roughly \$60 million.

Start Section 6-4 The Definite Integral

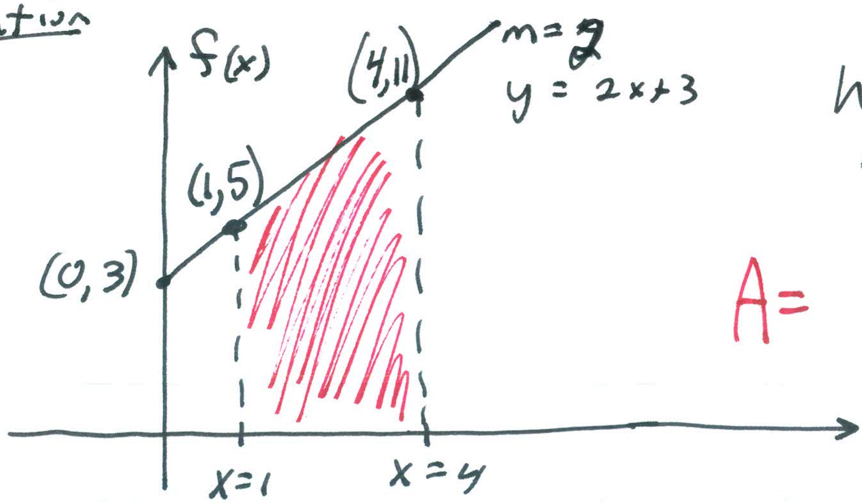
Basic Idea

Area under the graph of a function.

Example $f(x) = 2x+3$

Find the area under graph of f from $x=1$ to $x=4$.

Solution



We need the number that is the red shaded area.

$$L \cdot W + \tfrac{1}{2}bh$$

$$A = 3(5) + \tfrac{1}{2}(3)(6)$$

$$= 15 + 9$$

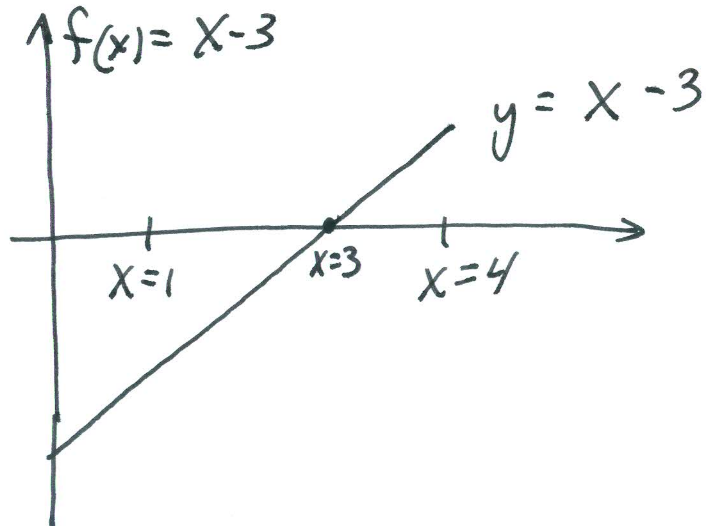
$$= 24$$

Notice that a better way of describing the area that we found would be

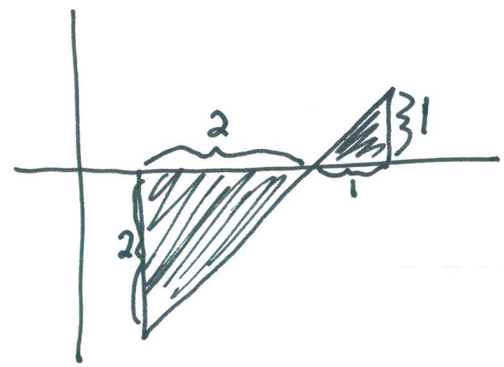
"the area between the graph of f and the x-axis, from $x=1$ to $x=4$."

What about a graph that crosses the x-axis?

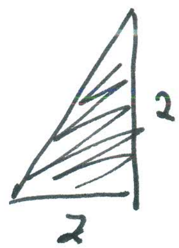
Example



Shade the area between graph of f and the x-axis, from $x=1$ to $x=4$



Find area of the shapes



$A = \frac{1}{2}(2)(2) = 2$

$A = \frac{1}{2}(1)(1) = \frac{1}{2}$

"Unsigned area" "under graph of f" from $x=1$ to $x=4$.

Unsigned area $= 2 + \frac{1}{2} = \frac{5}{2}$

"Signed Area" $= -2 + \frac{1}{2} = -\frac{3}{2}$

negative sign
because big triangle is
under the x-axis.

New terminology

The "Signed Area" under the graph of a function f from $x=a$ to $x=b$, is the sum of the ~~areas~~ areas of the ~~basic~~ regions between the graph of f and the axis, with a negative sign attached to the area of any region below the x-axis.

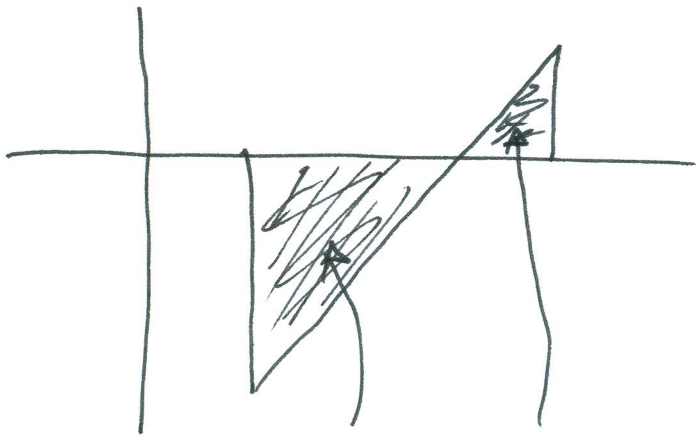
Symbol for Signed Area

$$A = \int_{x=a}^{x=b} f(x)\,dx$$ = signed area between graph of f and the x-axis, from $x=a$ to $x=b$.

= The Definite Integral of f from $x=a$ to $x=b$.

So for the function $f(x) = x+1$,

we can say that $A = \int_{x=1}^{x=4} f(x)\,dx = \int_{x=1}^{x=4} x+1\,dx = -\frac{3}{2}$



$A = \quad -2 \quad + \frac{1}{2}$

How do we find the "signed area" under graphs that are not straight lines?!

(We will talk more about this tomorrow (Tuesday)

Today, we will do a class drill about <u>estimating</u> the signed area.

<u>Class Drill 16</u>

"Left Sum" is the sum of the areas of rectangles that touch the graph of f at their left edges.

"Right Sum" is the sum of areas of rectangles that touch the graph of f at their right edges.

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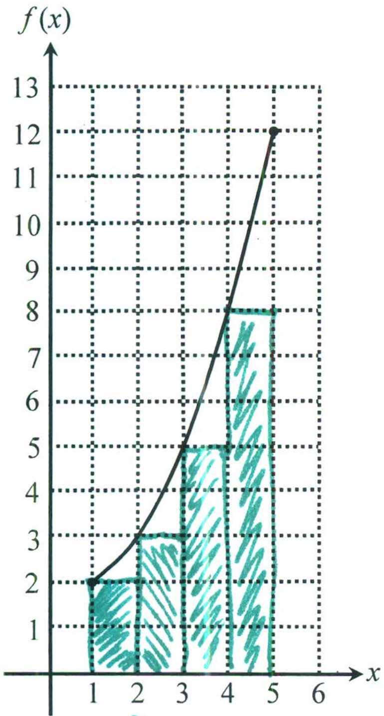
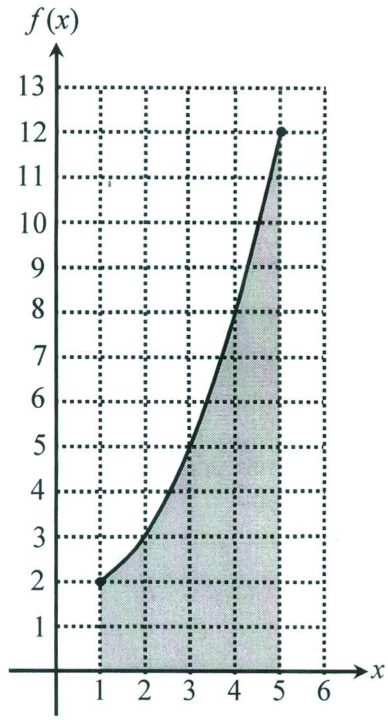
Class Drill 16: Estimating the Area Under a Graph Using Riemann Sums.

The goal is to estimate the shaded area in the middle figure. You will do this by finding the values of the Riemann sums L_4 and R_4. This will give you lower and upper bounds for the shaded area.

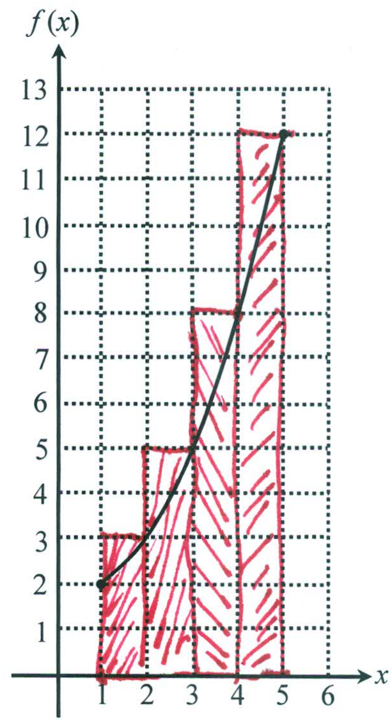


Left sum

L_4 ← 4 rectangles



$A = \int_1^5 f(x)dx$



R_4

(A) Draw in the rectangles for the left sum L_4. ✓

(B) Find the value of L_4. $(1\times 2) + (1\times 3) + (1\times 5) + (1\times 8) =$

$= 2+3+5+8 = 18$

(C) Draw in the rectangles for the right sum R_4.

(D) Find the value of R_4. $3+5+8+12 = 28$

(E) In the expression $L_4 \leq \int_1^5 f(x)dx \leq R_4$, replace the symbols L_4 and R_4 with the values from questions (B)and (D).

$$18 \leq \int_1^5 f(x)dx \leq 28$$