Day 40 is Tuesday, April 23, 2013

Your course Packet has the correct final exam time.

Wednesday, May 1, 10:10am – 12:10 pm in Morton 237

The Class web Page has the wrong time listed, but I will fix it today.

Section 7-2 Applications in Business + Economics

You do not need to read pages 421 + 422 in book, because we are not covering "Probability Density Functions."

Your reading starts on p. 423 "Continuous Income Stream"

General Idea of continuous income stream problems.

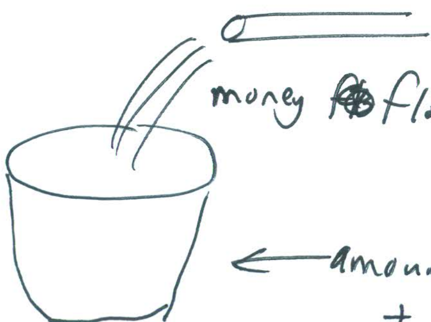
You have a big bucket for holding money.

The amount of money in the bucket at time t is $F(t)$

↑ capital F

The bucket starts out empty. So $F(0) = 0$.

Money starts flowing out of a pipe and into the Bucket.



money flow rate is $f(t)$ ← small f

← amount of money in bucket at time t is $F(t)$

Observe that $f(t) = F'(t)$

In "Flow Rate" problems, we know that $F(0)$
capital

and we are given $f(t)$ the flow rate.
small

We are asked to find $F(t)$ at some later time $t = b$. That is, we need to find $F(b)$

The pertinent equation is the Fundamental Theorem of Calculus.

$$\int_{t=a}^{t=b} f(t)\,dt = F(b) - F(a)$$

In the problems that we will study, the initial time is $t=0$. That is $a=0$.

$$\int_{t=0}^{t=b} f(t)\,dt = F(b) - F(0)$$

remember $F(0)=0$

$$\boxed{\int_{t=0}^{t=b} f(t)\,dt = F(b)}$$

Fundamental theorem of Calculus applied to problems where $F(0)=0$

The quantity $F(b)$ is called the "Future Value"

Also called the "Total Income"

Example 7-2 #22 (similar to #21)

Find the total income produced by a continuous income stream in the first 10 years if the flow rate is $f(t) = 3000$.

Solution

We know that $f(t) = 3000$ ("constant flow")

We know that $b = 10$ years.

We need to find $F(b) = F(10)$

$$F(10) = \int_{t=0}^{t=10} 3000\,dt = 3000t \Big|_{t=0}^{t=10} = 3000(10) - 3000(0)$$

$$= 30{,}000$$

Example 7-2 #24 (similar to #23)

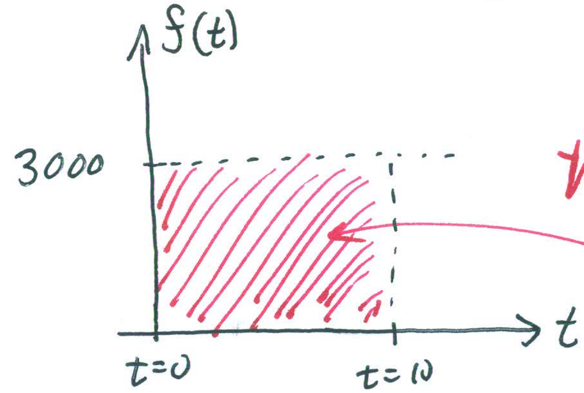
Illustrate the previous ~~xxx~~ problem with a graph.

(Book says "Interpret the results with both a graph and a description")

Solution We found

$$F(10) = \boxed{\int_{t=0}^{t=10} 3000\,dt} = 30000$$

This expression means "the ~~area~~ signed area under the graph of $y=3000$ from $t=0$ to $t=10$"



We found that the shaded area is 30,000

for coming examples it is useful to remember an antiderivative formula $\int e^{(cx)}\,dx = \dfrac{e^{(cx)}}{c} + K$

Example 7-2 #26 (similar to #25)

Find the total income produced by a continuous income stream in the first 2 years if the flow rate is

$$f(t) = 600e^{(.06t)}$$

Solution

$$\text{Total income} = \underset{\substack{\uparrow \\ \text{capital}}}{F}(2) = \int_{t=0}^{t=2} 600e^{(.06t)}\,dt$$

note $c = .06$

$$= 600\int_{t=0}^{t=2} e^{(.06t)}\,dt$$

$$= 600\,\frac{e^{.06t}}{.06}\Bigg|_{t=0}^{t=2}$$

note $\frac{600}{6} = 100$, so $\frac{600}{.6} = 1000$, so $\frac{600}{.06} = 10{,}000$

$$F(2) = 10{,}000\left(e^{.06t}\right)\Big|_{t=0}^{t=2}$$

$$= 10{,}000\left(e^{(.06)(2)} - e^{(.06)(0)}\right)$$

$$= 10{,}000\left(e^{.12} - e^{0}\right)$$

$$= 10{,}000\left(e^{.12} - 1\right)$$

$$\approx 1274.97$$

Example 7-2#28 (similar to #27)

Illustrate previous problem with a graph

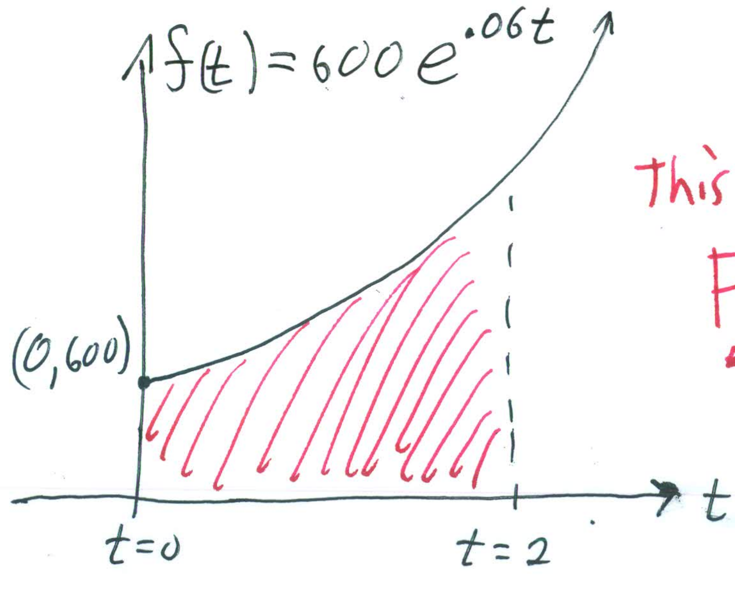
Solution We found a value for $F(2) = \int_{t=0}^{t=2} 600e^{.06t}\,dt$

capital

This quantity is the area under graph of $f(t) = 600e^{.06t}$

small f

from $t=0$ to $t=2$



This shaded area is

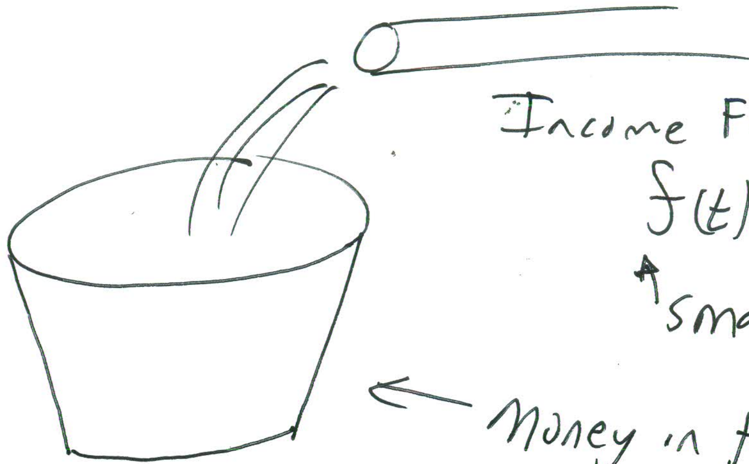
$F(2) = 10000(e^{.12}-1) \approx 1274.97$

capital

exact

1274.97

Future Value Problems



$F(t)$ = total amount of money in at time t

capital F

Bucket is empty initially, so $F(0) = 0$

Formula for Future Value is

$$FV = F(b) = \int_{t=0}^{t=b} f(t)\, e^{r(b-t)}\, dt$$

book uses capital T instead of letter b.

Future Value Example 7-2 #30 (similar to #29)

Starting at age 30, you deposit 2000/year into an IRA account. Treat the deposit as a continuous income stream. $f(t) = 2000$.

The IRA account earns 6% interest compounded continuously

(A) How much money will be in account 35 years later, when you retire at age 65?

Solution Notice $b = 35$ $r = .06$

$$FV = \int_{t=0}^{t=35} 2000\, e^{.06(35-t)}\, dt$$

$$= 2000 \int_{t=0}^{t=35} e^{.06(35-t)}\, dt$$

$$= 2000 \int_{t=0}^{t=35} \underbrace{e^{(.06)(35)}}_{\text{multiplicative constant}} e^{-.06t}\, dt$$

$$= 2000\, e^{(.06)(35)} \int_{t=0}^{t=35} e^{-.06t}\, dt$$

$$e^{.06(35-t)} = e^{(.06)(35)-(.06)(t)}$$

$$= e^{(.06)(35)} e^{-.06(t)}$$

$$e^{a+b} = e^a e^b$$

$$FV = 2000e^{(.06)35} \left. \frac{e^{-.06t}}{(-.06)} \right|_{t=0}^{t=35}$$

⋮ used wolfram

≈ 238872

<u>Question (B)</u> How much of the amount in (A) is from interest?

<u>Solution</u> A total of $(35 \text{ years}) \times \left(\frac{\$2000}{\text{year}}\right) = \$70{,}000$ came out of the pipe. The rest was interest. So interest is

$\$238{,}872 - 70{,}000 = \$168{,}872$ in interest