

**Reference 3: Facts About Limits from Section 3-1**

Suppose that  $\lim_{x \rightarrow c} f(x) = L$  and  $\lim_{x \rightarrow c} g(x) = M$  exist, where  $L$  and  $M$  are real numbers. Then

<b>Theorem 2.1:</b>	$\lim_{x \rightarrow c} k = k$
<b>Theorem 2.2:</b>	$\lim_{x \rightarrow c} x = c$
<b>Theorem 2.3:</b>	$\lim_{x \rightarrow c} [f(x) + g(x)] = \lim_{x \rightarrow c} f(x) + \lim_{x \rightarrow c} g(x) = L + M$
<b>Theorem 2.4:</b>	$\lim_{x \rightarrow c} [f(x) - g(x)] = \lim_{x \rightarrow c} f(x) - \lim_{x \rightarrow c} g(x) = L - M$
<b>Theorem 2.5:</b>	If $k$ is a constant, then $\lim_{x \rightarrow c} kf(x) = k \lim_{x \rightarrow c} f(x) = kL$ .
<b>Theorem 2.6:</b>	$\lim_{x \rightarrow c} (f(x) \cdot g(x)) = (\lim_{x \rightarrow c} f(x)) \cdot (\lim_{x \rightarrow c} g(x)) = L \cdot M$
<b>Theorem 2.7:</b>	If $\lim_{x \rightarrow c} g(x) = M \neq 0$ , then $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)} = \frac{L}{M}$
<b>Theorem 2.8:</b>	If $n$ is a positive odd integer, or if $n$ is a positive even integer and $\lim_{x \rightarrow c} f(x) = L > 0$ , then $\lim_{x \rightarrow c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow c} f(x)}$ .
<b>Theorem 3:</b>	If $f$ is a polynomial, then $\lim_{x \rightarrow c} f(x) = f(c)$ . If $r$ is a rational function and $c$ is in the domain of $r$ , then $\lim_{x \rightarrow c} r(x) = r(c)$ .
<b>Definition:</b>	If $\lim_{x \rightarrow c} f(x) = L = 0$ and $\lim_{x \rightarrow c} g(x) = M = 0$ , then then $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$ is said to have the <i>indeterminate form</i> $\frac{0}{0}$ . In this case, Theorem 2.7 cannot be used to determine the limit.
<b>Theorem 4:</b>	If $\lim_{x \rightarrow c} f(x) = L \neq 0$ and $\lim_{x \rightarrow c} g(x) = M = 0$ , then $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$ does not exist according to the Definition of the Limit found in Section 3-1. (Remark: In Section 3-2, the Definition of Limit gets expanded, with the result that in some cases, we will say that the limit is infinity or negative infinity and will write the symbol $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \infty$ or $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = -\infty$ . But this will not always be the case. That is, sometimes the limit will not exist even with the expanded Definition of Limit.)
<b>One x Rule (not in book)</b>	If $f$ and $g$ are functions whose y-values differ at only one x-value, $c$ (that is, the y-values are the same for $x \neq c$ but differ at $x = c$ ), then $\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} g(x)$ .