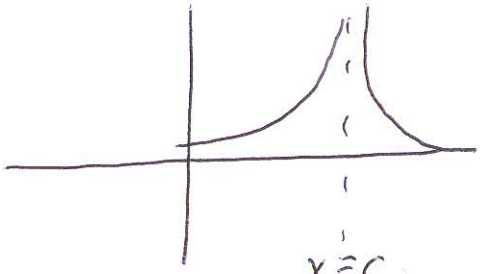
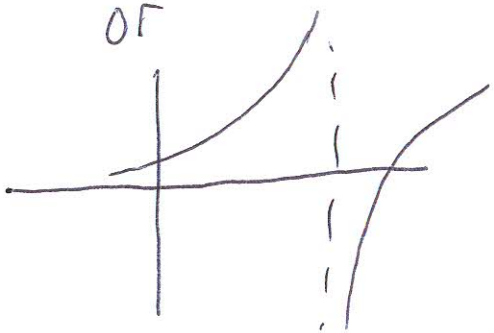
Friday, August 30, 2013 (Day 4)

Summary of Limits Involving Infinity (Graphical Approach)

Graph That Has Vertical Asymptote at $x = c$ $\Longleftrightarrow$ (limit behavior at $x = c$) (infinite limits)



or

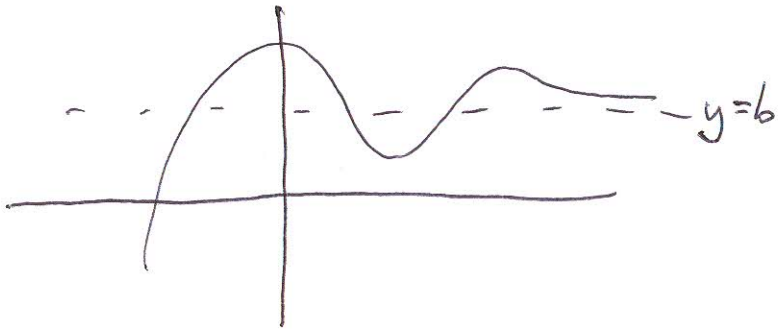


$\lim_{x \to c^-} f(x) = \pm\infty$

$\lim_{x \to c^+} f(x) = \pm\infty$

$\lim_{x \to c} f(x) = \pm\infty$ or DNE

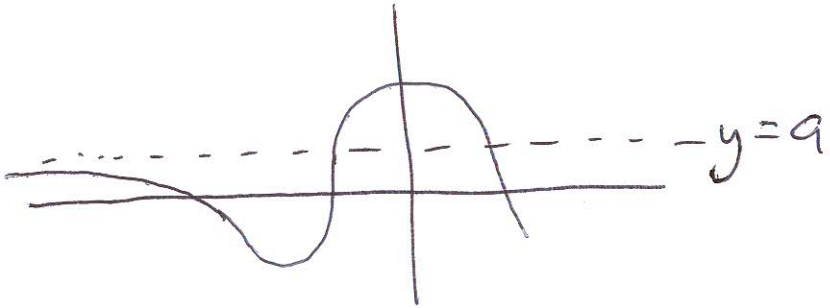
Graph with Horiz asymptote on right at $y=b$



$\Longleftrightarrow$ limit behavior (limit <u>at</u> infinity)

$$\lim_{x \to \infty} f(x) = b$$

Horizontal Asymptote on <u>left</u> at $y=a$

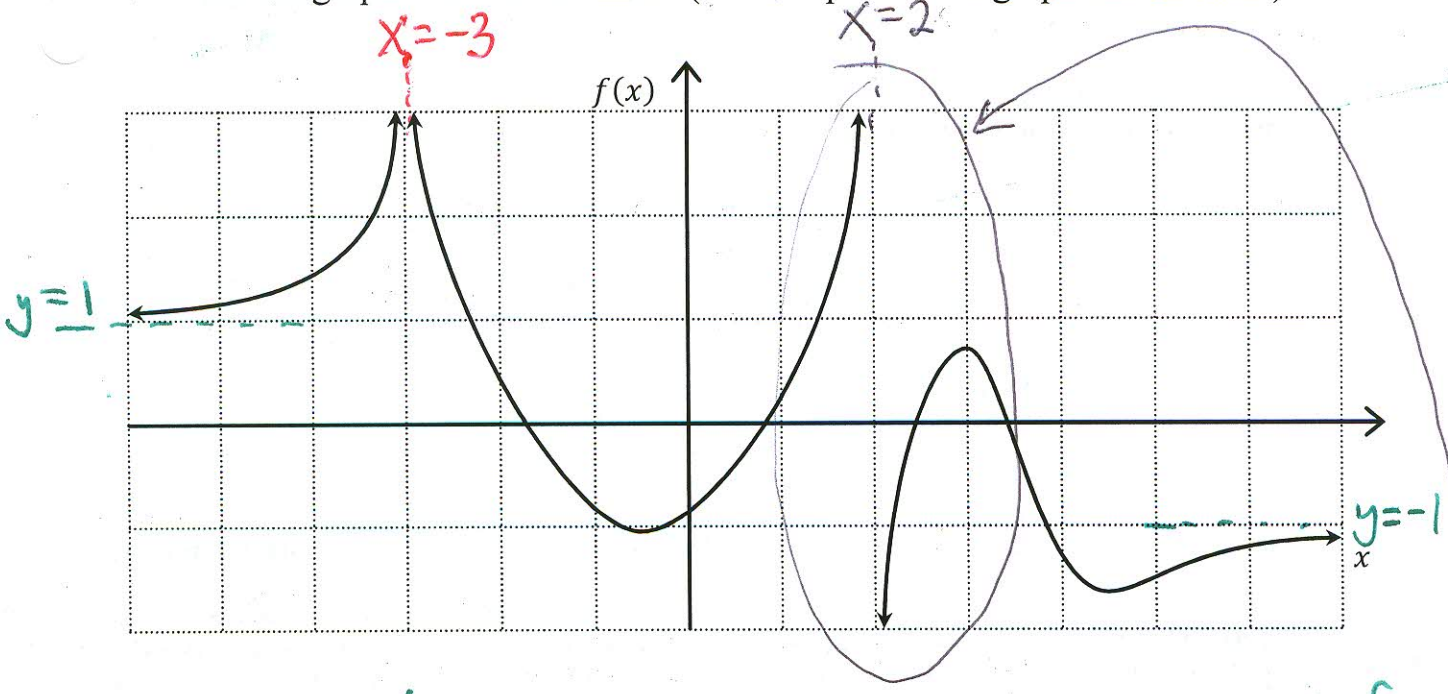


$\Longleftrightarrow \lim_{x \to -\infty} f(x) = a$

Now Do Class Drill 2

Class Drill 2: Limits Involving Infinity

Use the graph to fill in the table. (Extra copies of the graph are on back.)



(A) $\lim_{x\to-\infty} f(x) =$ 1 because horiz asymptote on left at $y=1$.

(B) $\lim_{x\to-3} f(x) =$ ∞ because symmetric vertical asymptote going up at $x=-3$

(C) $\lim_{x\to2^-} f(x) =$ ∞

(D) $\lim_{x\to2^+} f(x) =$ $-\infty$

(E) $\lim_{x\to2} f(x) =$ DNE

} asymmetric vertical asymptote at $x=2$ (up on left, down on right.)

(F) $\lim_{x\to\infty} f(x) =$ -1 because horiz asymptote on right at $y=-1$

Now take an Analytic approach to limits involving infinity.

Start by considering limits at infinity

That is, $\lim_{x \to \infty} f(x)$ or $\lim_{x \to -\infty} f(x)$

where $f(x)$ is described by a formula.

We will be interested in these three functions

$$f(x) = \frac{9x^2 - 90x + 189}{2x^2 - 24x + 70}$$

$$g(x) = \frac{9x^2 - 90x + 189}{2x^3 - 24x^2 + 70x}$$

$$h(x) = \frac{9x^3 - 90x^2 + 189x}{2x^2 - 24x + 70}$$

Identify Leading Terms

find $\lim_{x \to \infty} f(x)$

$$= \lim_{x \to \infty} \frac{9x^2 - 90x + 189}{2x^2 - 24x + 70}$$

$$= \lim_{x \to \infty} \frac{9x^2}{2x^2}$$

the limit is the same as the limit with just the leading terms

$$= \lim_{x \to \infty} \frac{9}{2}$$

$$= \frac{9}{2}$$

This tells us that the graph of f will have a horizontal asymptote on right at $y = \frac{9}{2}$

Find $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \dfrac{9x^2 - 90x + 189}{2x^3 - 24x^2 + 70x}$

Identify leading terms

$= \lim_{x\to\infty} \dfrac{9x^2}{2x^3}$

$= \lim_{x\to\infty} \dfrac{9}{2x}$

$= 0$

Conclusion: graph of g has horizontal asymptote on right at $y = 0$.