Wednesday, September 18, 2013 (Day 14)

Be sure to swipe your I.D.

Exam Friday I will be checking your ID's during the exam. (sorry!)

Section 3-7 Marginal Analysis

Business Terminology

Demand, Price, Revenue, Cost, Profit
from Reference 5 in Course Packet (page 4)
We will not cover "Average Quantities" this semester.

Marginal Quantities

"Marginal Quantity" means "The Derivative of Quantity"

Example: "Marginal Revenue" means $R'(x)$. etc.

Examples involving a company with

Revenue function $R(x) = 5X - .02X^2$

Cost function $C(x) = 145 + 1.1X$

(A) Find the marginal cost function.

Solution

$$\text{Marginal Cost} = C'(x)$$

$$= \frac{d}{dx}\left(145 + 1.1x\right)$$

identify multiplicative constants

$$= 145\left(\frac{d}{dx}1\right) + 1.1\left(\frac{d}{dx}x\right)$$

$$= 145(0) + 1.1(1)$$

$$= 1.1$$

(B) Find the Marginal Revenue

Solution

Marginal Revenue $= R'(x)$

$$= \frac{d}{dx}(5x - .02x^2)$$

$$= 5\left(\frac{d}{dx}x\right) - .02\left(\frac{d}{dx}x^2\right)$$

$$= 5(1) - .02(2x)$$

$$= 5 - .04x$$

(c) Find the marginal Profit

Solution

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$P(x) = R(x) - C(x)$$

$$\text{Marginal profit} = P'(x)$$

$$= \frac{d}{dx}\left(P(x)\right)$$

$$= \frac{d}{dx}\left(R(x) - C(x)\right)$$

$$= \frac{d}{dx}R(x) - \frac{d}{dx}C(x)$$ used sum rule

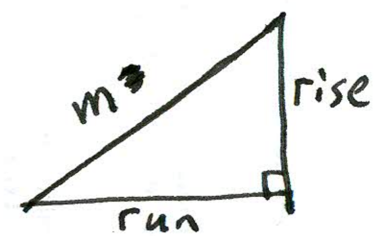
$$= R'(x) - C'(x)$$

marginal profit $= (5 - .04X) - (1.1)$

$= 3.9 - .04X$

Estimation Problems

Review slope calculations



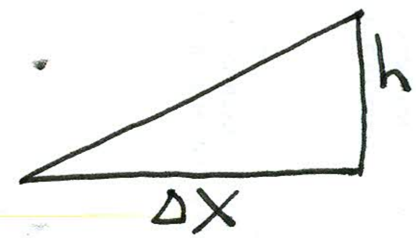
slope $m = \frac{\text{rise}}{\text{run}}$

trick: multiply both side of this equation by "run"

$$m \cdot \text{run} = \left(\frac{\text{rise}}{\cancel{\text{run}}}\right) \cdot \cancel{\text{run}}$$

$$m \cdot \text{run} = \text{rise}$$

use different terminology



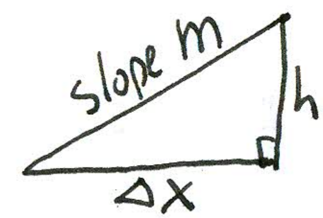
$$m = \frac{h}{\Delta x}$$

multiply both sides by Δx

$$m \cdot \Delta x = h$$

turn this around

$$h = m \cdot \Delta x$$



Change in y-value for a function

Suppose we have a function called f.

Consider changing the x value from x_1 to x_2.

What is the change in x? Answer: $\Delta x = x_2 - x_1$

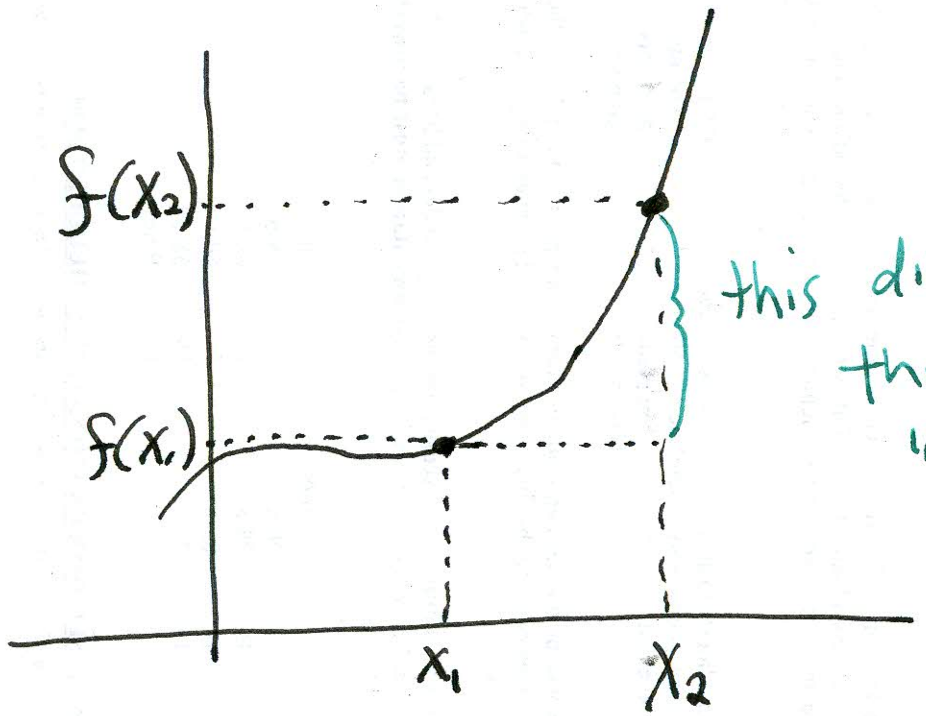
What are the corresponding y-values?

Answer $f(x_1)$ and $f(x_2)$

What is the change in y-value?

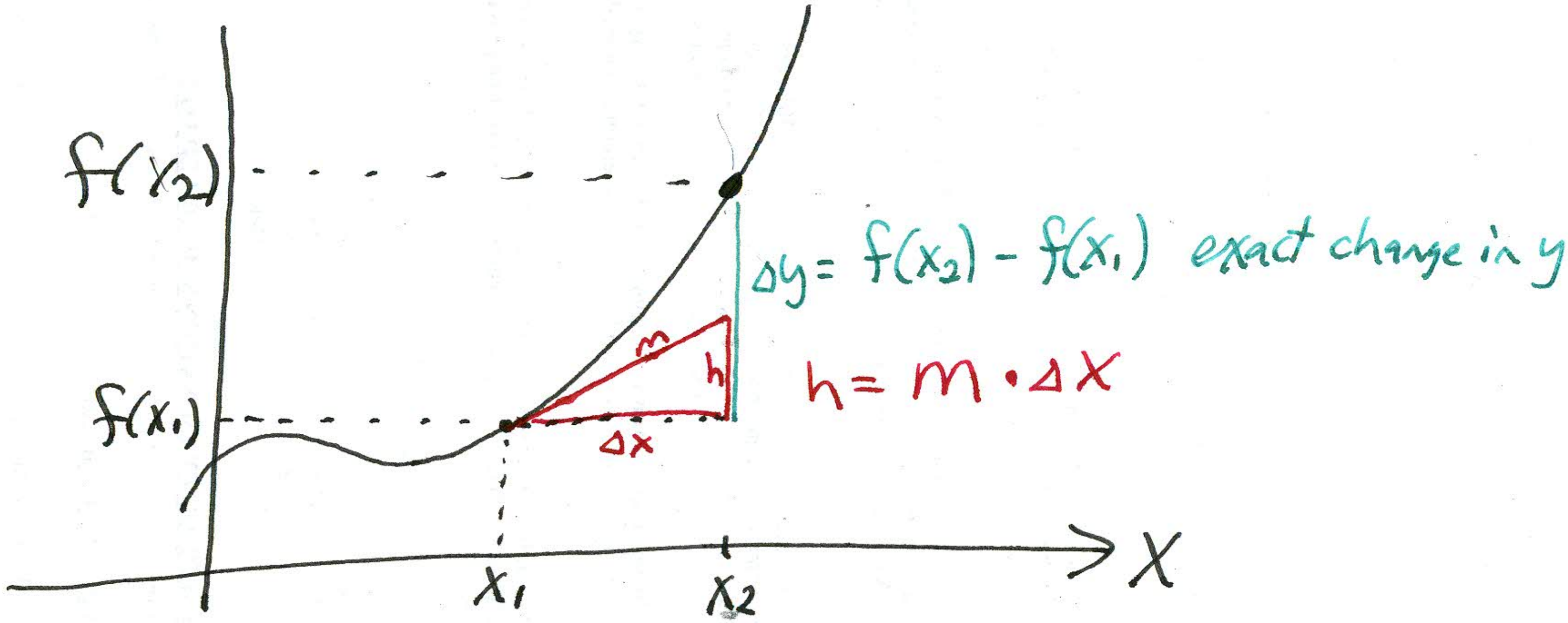
Answer: $\Delta y = f(x_2) - f(x_1)$

Consider Change in y on a graph



this distance is Δy.
this is the "exact change in y"

Superimpose a little triangle with hypotenuse that is <u>tangent</u> to graph at $x = x_1$



$$h = m \cdot \Delta x \quad = f'(x_1) \cdot \Delta x$$

↑ slope of the line tangent to graph of f at x_1

So we have these two quantities

green quantity: $\Delta y = f(x_2) - f(x_1)$ = exact change in y

red quantity: $h = f'(x_1) \cdot \Delta x$ = height of little red ~~sq.~~ triangle.

The red quantity is called the approximate change in y.

exact change in y is $\Delta y = f(x_2) - f(x_1)$

approximate change in y is $h = f'(x_1) \cdot \Delta x$.

Relationship: Exact Change $\approx$ Approximate Change

$$\Delta y \approx h$$

$$f(x_2) - f(x_1) \approx f'(x_1) \cdot \Delta x$$

Example

3 − 7 #26 A company makes guitars

The exact cost of building a batch of x guitars is

$$C(x) = 1000 + 100x \cancel{+} - 0.25x^2 \text{ dollars}$$

minus

(A) If Batch size changes from $x_1 = 50$ to $x_2 = 51$ guitars, what is the exact change in cost?

Answer exact change $= \Delta y = \Delta C = C(51) - C(50)$

$= \$74^{\underline{75}}$

result using calculator

(B) Use marginal cost to approximate that change.

Solution

approximate change $= h = C'(50) \cdot \Delta x$