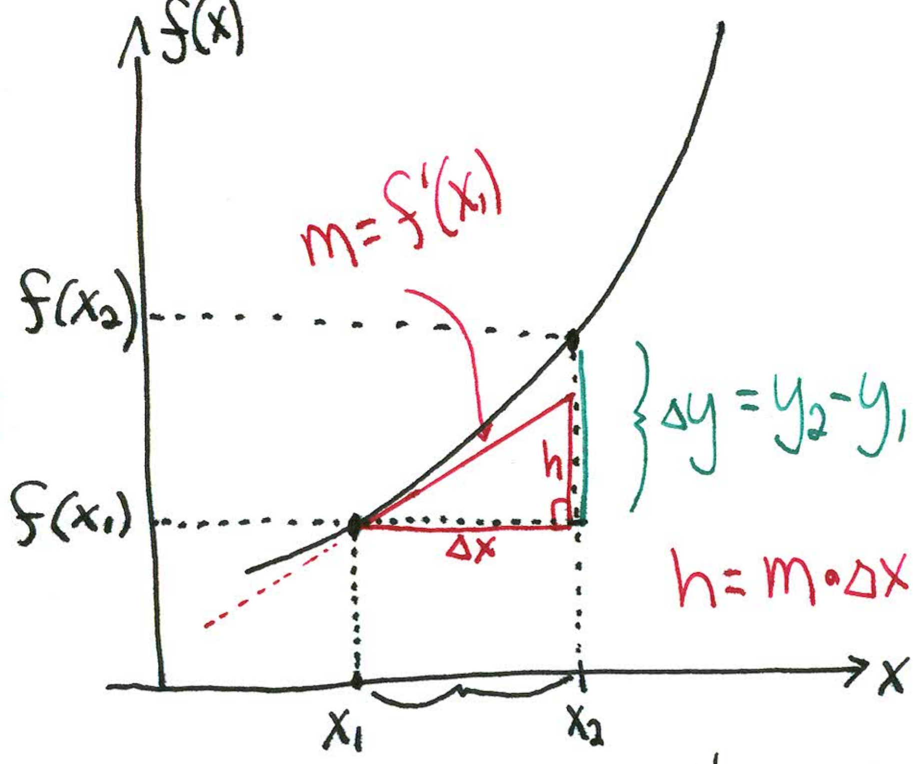
Monday, September 23, 2013 (Day 16)

Leftovers from Section #3-7 Marginal Analysis

Recall this picture from Wednesday, Sept 18.

Some function called f. Considering what happens when the input changes from $x = x_1$ to $x = x_2$.

the line tangent to graph of f at $x = x_1$ has slope $m = f'(x_1)$



$\Delta y = y_2 - y_1 = f(x_2) - f(x_1) =$ exact change in y ✓

$h = m \cdot \Delta x = f'(x_1) \cdot \Delta x =$ approximate change in y.

$\Delta x = x_2 - x_1 =$ change in x

<u>Main issues:</u>

(1) We are usually interested in knowing the value of Δy (the exact change in y)

(2) But the value of h is often very close to the value of Δy. That's why h is called the "approximate change"

(3) And often, Δy is difficult to calculate but h is <u>easy</u> to calculate.

So we often will compute h to get an approximate value for the change in y.

Return to wednesday's example

A company makes guitars.

The ~~de~~ Demand x is the number of guitars made (and sold) each week.

The Cost $C(x)$ of producing x guitars is $C(x) = 1000 + 100x - .25x^2$

(A) If batch size changes from $x_1 = 50$ to $x_2 = 51$, What is the exact change in cost of producing a batch of guitars?

(Book wording: Find the exact cost of ~~pr~~ producing the 51^{st} guitar.)

Solution

We need to find

$\Delta C = \Delta y =$ exact change in cost

$\cancel{= f(51) - f(}$

$= C(51) - C(50)$

Use computer: $C(50) = 5375$ ← used calculator

$C(51) = 5449.75$ ←

$$\Delta C = \Delta y = C(51) - C(50) = \begin{array}{r} 5449.75 \\ -\ 5375.00 \\ \hline \$74.75 \end{array}$$

~~(B)~~ (B) If the Batch Size changes from $x_1 = 50$ to $x_2 = 51$ guitars, use the Marginal Cost function to find an approximate value for the change in Cost of producing a batch of guitars. That is, use Marginal Cost to find an approximation for ΔC.

(Book Wording: Use the Marginal Cost to approximate the cost of producing the 51^{st} guitar.)

Solution

Approximate Change $= h = \underbrace{C'}_{\text{marginal cost!!}}(X_1) \cdot \Delta X$ from page ① of these notes

We need to find the value of h.

$$\Delta X = X_2 - X_1 = 51 - 50 = 1$$

To find $C'(X_1)$, we need to find $C'(X)$

Substitute in $X_1 = 50$

<u>Get $C'(x)$</u>

$$C(x) = 1000 + 100x - .25x^2$$

$$C'(x) = \frac{d}{dx}\left(1000 + 100x - .25x^2\right)$$

$$= 1000\left(\frac{d1}{dx}\right) + 100\left(\frac{dx}{dx}\right) - (.25)\left(\frac{dx^2}{dx}\right)$$

$$= 1000(0) + 100(1) - (.25)(2x)$$

$$C'(x) = 100 - .5x$$

$$C'(x_1) = C'(50) = 100 - .5(50) = 75$$

So approx change $= h = C'(51) \cdot \Delta x = 75(1) = 75$

Summary

exact change $= \Delta C = \$74^{\underline{75}}$

approximate change $= h = \$75$

exact change $\approx$ approximate change

$$\Delta C \approx h$$

That is

$$\Delta C \approx h = \$\, C'(50) \cdot \Delta x = (75)(1)$$

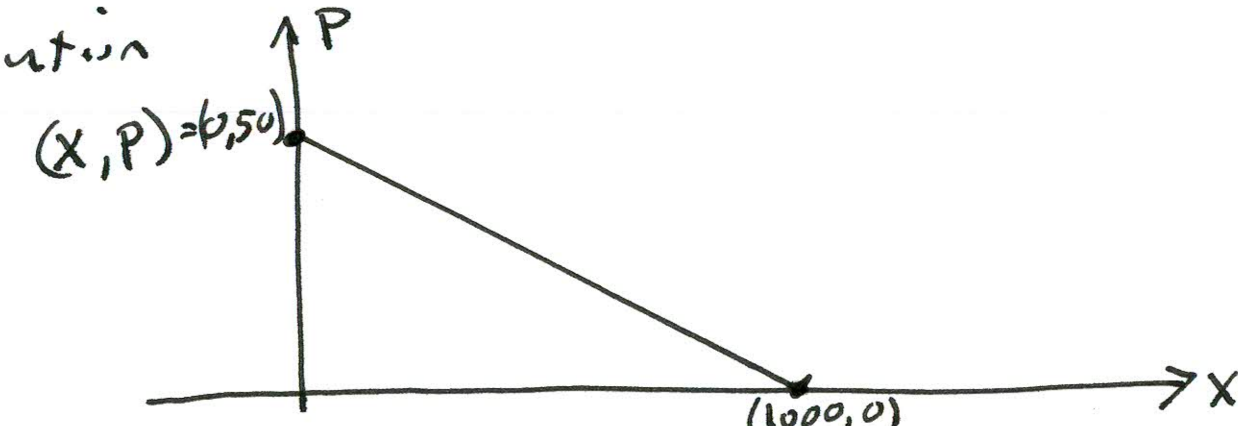
Another Example

Price p ← small p and Demand x for a particular item are related by the following equation

$$p = -\frac{x}{20} + 50 \quad \text{with Domain } 0 \le x \le 1000$$

(A) graph the price demand equation

Solution



Coordinates of x-intercept?? Set p=0 and solve for x

$$0 = -\frac{x}{20} + 50$$

$$\frac{x}{20} = 50$$

$$x = (20)(50) = 1000$$

x-intercept at $(x, p) = (1000, 0)$

(B) Find the Revenue Function and determine its domain

Solution: Revenue = number of items sold • selling price per item

= demand • price

$$R = x \cdot p$$

But if we substitute in the price function $p(x)$ a function of x

$$\left(\begin{aligned} p &= -\frac{x}{20} + 50 \\ p(x) &= -\frac{x}{20} + 50 \end{aligned} \right)$$

We end up with a Revenue function of the variable x.

$$R(x) = x \cdot p(x) = x\left(-\frac{x}{20} + 50\right)$$

$R(x) = x\left(-\frac{x}{20} + 50\right)$ Revenue function

The Domain of the Revenue Function $R(x)$

is also $0 \le x \le 1000$

because we used the price function

$$P(x) = -\frac{x}{20} + 50$$

to build $R(x)$, and the price function

is only valid for $0 \le x \le 1000$.

(C) Find the Marginal Revenue at a Production level of 400 items and interpret the results.

Solution

Translation: Find $R'(400)$ and interpret the results.

Strategy: Find $R'(x)$
Substitute in $x = 400$.

$$R(x) = x\left(-\frac{x}{20} + 50\right)$$

rewrite a sum of constants & power functions

$$= -\frac{x^2}{20} + 50x$$

$$R(x) = \left(-\frac{1}{20}\right)x^2 + 50x$$

Find the Derivative

$$R'(x) = \frac{d}{dx}\left(\left(-\frac{1}{20}\right)x^2 + 50x\right)$$

$$= \left(-\frac{1}{20}\right)\frac{d}{dx}x^2 + 50\left(\frac{d}{dx}x\right)$$

$$= \left(-\frac{1}{20}\right)(2x) + 50(1)$$

$$R'(x) = -\frac{x}{10} + 50 \quad \text{Marginal Revenue}$$

$$R'(400) = -\frac{400}{10} + 50 = -40 + 50 = 10$$

This is the marginal Revenue at a production level of 400 units.

But what is the "interpretation" of this? Discuss tomorrow.