Monday, September 30, 2013 (Day 20)

- Pick up your graded work
- Swipe your I.D.
- Quiz 5 on Friday: Study the Textbook Examples and do the Matched Problems!

Common Mistake on Quiz 4

Replacing $2 \cdot 3^{(x)}$ with $6^{(x)}$. This is incorrect.

Look at the left expression. Two ~~things are~~ operations are indicated. Multiplication + exponent.

The exponent gets done first.

If you replace $2 \cdot 3^{(x)}$ with $6^{(x)}$, it means that you have done the multiplication first. (order of operations mistake)

A couple more examples and Class Drill on Section 4-2 concepts.

Example #4 $f(x) = \underbrace{12\ln(13)}_{\text{constant function}}$ find $f'(x)$.

Solution: $f'(x) = 0$.

Example #5 $f(x) = 12\ln(13x)$ find $f'(x)$.

Solution: must first rewrite $f(x)$ so that we can use Rule#1 or Rule#2 from Friday

Step 1 rewrite $f(x) = 12\ln(13x) = 12(\ln(13) + \ln(x))$

$\ln(ab) = \ln(a) + \ln(b)$

$$f(x) = 12\ln(13) + 12\ln(x)$$

Step 2 Find the derivative

example#4 example#1

$$f'(x) = \frac{d}{dx}12\ln(13) + \frac{d}{dx}12\ln(x) = 0 + \frac{12}{x} = \frac{12}{x}$$

Work on Class Drill 6

Class Drill 6 Derivatives of Functions Containing Logarithms

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(A) Let $f(x) = 12\ln\left(\frac{13}{x}\right)$. Find $f'(x)$. (Start by rewriting f using a rule of logarithms.)

$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

Step 1: rewrite $f(x) = 12\ln\left(\frac{13}{x}\right) = 12(\ln(13) - \ln(x))$

$= 12\ln(13) - 12\ln(x)$

$f'(x) = 0 - \frac{12}{x}$ using results of examples #1 and #4

$f'(x) = \frac{-12}{x}$

(B) Let $f(x) = 12\ln(x^{13})$. Find $f'(x)$. (Start by rewriting f using a rule of logarithms.)

$\ln(a^b) = b\ln(a)$

Step 1: rewrite $f(x) = 12\ln(x^{13}) = 12 \cdot 13 \cdot \ln(x)$

Step 2: derivative $f'(x) = \frac{d}{dx}(12 \cdot 13)\ln(x)$ multiplicative constants

$= (12 \cdot 13)\frac{d}{dx}\ln(x) = (12 \cdot 13)\left(\frac{1}{x}\right) = \frac{12 \cdot 13}{x}$

(C) Let $f(x) = 12x\ln(13)$. Find $f'(x)$.

Step 1: rewrite f with multiplicative constants in front

$f(x) = (12)x(\ln(13)) = 12 \cdot \ln(13) \cdot x$

Step 2: Derivative $f'(x) = \frac{d}{dx}(12\ln(13)) \cdot x = 12\ln(13)\frac{d}{dx}x = 12\ln(13)(1)$

$f'(x) = 12\ln(13)$

(D) The goal is to find the equation of the line tangent to the graph of the function

$$f(x) = 5 + \ln(x^3)$$

at the point where $x = e^2$.

Remember that the approach is to build the general form of the equation for the tangent line (in point-slope form):

$$(y - f(a)) = f'(a) \cdot (x - a)$$

Question (D) continues on back. ➔

Get Parts

Identify the number a.

$a = e^2$ (this is the x-coord of the point of tangency.)

$\ln(e^x) = x$

Find $f(a)$.

$$f(a) = 5 + \ln((e^2)^3) = 5 + \ln(e^6) = 5 + 6 = 11$$

(this is the y-coord. of the point of tangeny.)

Find $f'(x)$. Hint: Start by rewriting f using a rule of logarithms.

Step 1 Rewrite $f(x) = 5 + \ln(x^3) = 5 + 3\ln(x)$

Step 2 Take derivative

constant function

$$f'(x) = \frac{d}{dx}(5 + 3\ln(x)) = \frac{d5}{dx} + 3\frac{d\ln(x)}{dx} = 0 + 3\left(\frac{1}{x}\right) = \frac{3}{x}$$

multiplicative constant

Find $f'(a)$.

$m = f'(a) = \frac{3}{e^2}$ = slope of the tangent line

Substitute Parts Into the General Tangent Line Equation

$$(y - 11) = \frac{3}{e^2}(x - e^2)$$

Convert the Equation to Slope Intercept Form

$$y - 11 = \left(\frac{3}{e^2}\right)x - \left(\frac{3}{e^2}\right)e^2 = \left(\frac{3}{e^2}\right)x - 3$$

add 11 to both sides

$$y = \left(\frac{3}{e^2}\right)x + 8$$

Section 4-3 Derivatives of Products + Quotients

The product Rule Used for taking the derivative of a product of functions. $f(x) \cdot g(x)$

~~The obvious thing~~ is wrong!!!

$$\cancel{\frac{d}{dx}\left(f(x) \cdot g(x)\right) = \frac{df(x)}{dx} \cdot \frac{dg(x)}{dx}}$$

The Product Rule:

$$\boxed{\frac{d}{dx}\left(f(x) \cdot g(x)\right) = \left(\frac{df(x)}{dx}\right) g(x) + f(x)\left(\frac{dg(x)}{dx}\right)}$$

Example #1 $f(x) = (-3x^2 + 5x - 7) \cdot (3x - 2)$

Find $f'(x)$ using the product rule.

Solution

$$f'(x) = \left(\frac{d}{dx}\left(-3x^2 + 5x - 7\right)\right) \cdot (3x-2) + \left(-3x^2+5x-7\right)\left(\frac{d}{dx}\, 3x-2\right)$$ Product Rule

$$= (-6x + 5 + 0)(3x-2) + (-3x^2 + 5x - 7)(3+0)$$ derivatives

$$= (-6x+5)(3x-2) + (-3x^2+5x-7)(3)$$

$$= \underbrace{-18x^2} + \underbrace{12x + 15x} - \underbrace{10} - \underbrace{9x^2} + \underbrace{15x} - \underbrace{21}$$

$$= -27x^2 + 42x - 31$$

simplifying

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Example #2 $f(x) = (-3x^2 + 5x - 7) \cdot e^{(x)}$ Find $f'(x)$

Solution

$$f'(x) = \left(\frac{d}{dx}(-3x^2+5x-7)\right) \cdot e^{(x)} + (-3x^2+5x-7)\frac{d}{dx}e^{(x)}$$ product rule

$$= (-6x+5)e^{(x)} + (-3x^2+5x-7)(e^{(x)})$$ derivatives

factor: $ac + bc = (a+b)c$

$$= ((-6x+5) + (-3x^2+5x-7))e^{(x)}$$ factored out the $e^{(x)}$

$$= (-3x^2 - x - 2)e^{(x)}$$ simplified