Wednesday, October 23, 2013 (Day 34)

- Swipe your I.D.
- Pick up your graded work
- Remember Quiz 7 on Friday: Study book examples
- Resume work on example from end of class yesterday

Resuming example finding absolute extrema of $f(x) = x^4 - 6x^2 + 5$

Finding the corresponding y-values

$f(-3) = (-3)^4 - 6(-3)^2 + 5 = 81 - 6(9) + 5 = 81 - 54 + 5 = 32$

$f(-\sqrt{3}) = (-\sqrt{3})^4 - 6(-\sqrt{3})^2 + 5 = 9 - 6(3) + 5 = 9 - 18 + 5 = -4$

$f(0) = 5$

$f(\sqrt{3}) = -4$ Same as $f(-\sqrt{3})$ because f is an <u>even</u> function.

$f(2) = (2)^4 - 6(2)^2 + 5 = 16 - 6(4) + 5 = 16 - 24 + 5 = -3$

Fill in table from yesterday

important x-values		corresponding y-values
$x = -3$	endpoint	$f(-3) = 32$
$x = -\sqrt{3}$	critical	$f(-\sqrt{3}) = -4$
$x = 0$	critical	$f(0) = 5$
$x = \sqrt{3}$	critical	$f(\sqrt{3}) = -4$
$x = 2$	endpoint	$f(2) = -3$

Step 4 Conclusion

The absolute max is $y = 32$ and it occurs at $x = -3$.

The absolute min is $y = -4$ and it occurs at $x = -\sqrt{3}$ and $x = \sqrt{3}$.

<u>A related example</u> Find the absolute extrema of

$f(x) = x^4 - 6x^2 + 5$ on the interval $[-1, 2]$

<u>Solution</u> Same ~~exam~~ function as previous example but different interval.

<u>Step 1</u> observe the interval $[-1,2]$ is closed and the function is continuous on the interval.

<u>Step 2</u> The critical values of $f(x)$ are $x = -\sqrt{3}$, $x=0$, $x=\sqrt{3}$.

Note: $\sqrt{3} \approx 1.732$ so $x=\sqrt{3}$ is in interval

but $x = -\sqrt{3}$ is not in interval

<u>Step 3</u>

list of important x-values		corresponding y-values
$x=-1$	endpoint	$f(-1) = (-1)^4 - 6(-1)^2 + 5 = 1 - 6(1) + 5 = 0$
$x=0$	critical	$f(0) = 5$
$x=\sqrt{3}$	critical	$f(\sqrt{3}) = -4$
$x=2$	endpoint	$f(2) = -3$

from previous example

<u>Step 4</u> Conclude the absolute max is $y=5$ and it occurs at $x=0$

the absolute min is $y=-4$ and it occurs at $x=\sqrt{3}$.

Now Consider what might happen when the interval is not closed, or when the function f is not continuous. The Extreme Value Theorem does not guarantee anything in this case.

Work on Class Drill 14

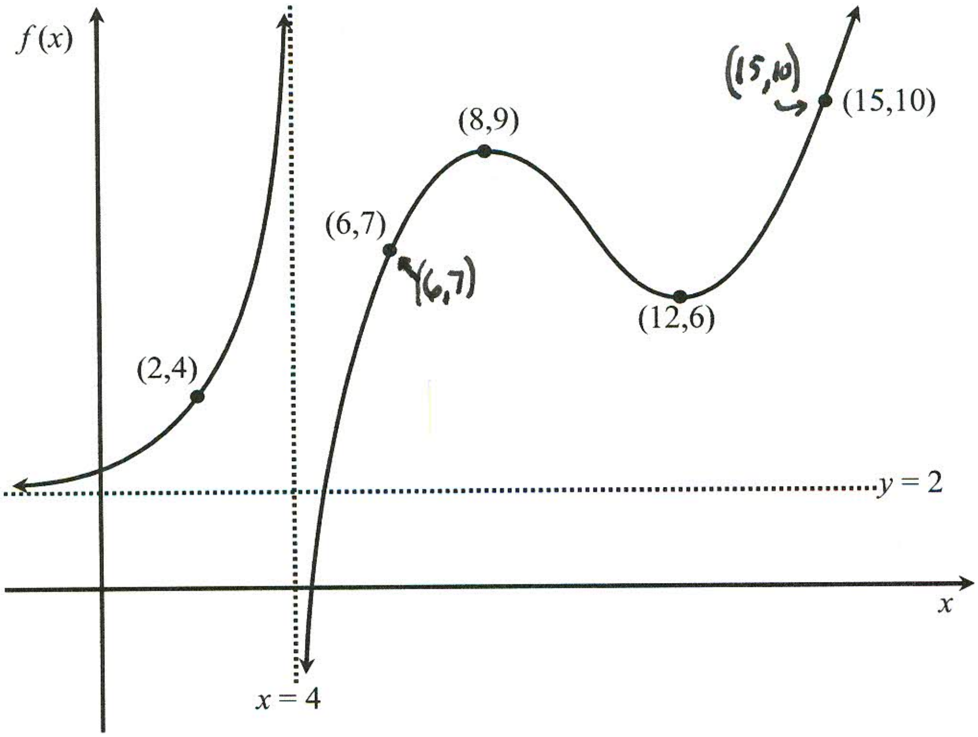
(Graphical Examples where Extreme Value Theorem Does not apply.)

5

Class Drill 14: Relative and Absolute Extrema

The *Extreme Value Theorem* says that if a function f is continuous on a closed interval $[a,b]$, then f will have both an absolute maximum and an absolute minimum on that interval. In this drill, you investigate what can happen when f is not continuous or the interval is not closed.

The graph of a function f is shown at right. Fill in the table below.



Interval	Relative Maxima in that interval	Relative Minima in that interval	Absolute Max in that interval	Absolute Min in that interval
[6,15]	y=9 y=10	y=7 y=6	y=10	y=6
not closed (6,15)	y=9	y=6	none	y=6
not closed (8,15)	none	y=6	none	y=6
f not continuous [2,12]	y=9	y=4 y=6	none	none
(2,12)	y=9	none	none	none
(4,∞)				

Now consider analytic examples where the extreme value theorem does not apply.

Example Find all absolute extrema of $f(x) = x^4 - 6x^2 + 5$ on the interval $(-\infty, \infty)$.

Solution

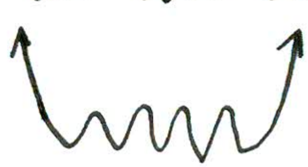
Do we have a continuous function? yes (polynomial)

Do we have a closed interval? no

So we are not guaranteed any absolute max or min.

Consider "end behavior" of the graph of f.

f has degree 4, with positive leading coefficient. So graph goes up on both ends. (See Reference 2 on p. 1 of Course Packet.



There won't be a highest point.
But there will be a lowest point.

So there will be an absolute min but not an absolute max.

Where do we look for the absolute min?!?

Theorem 2 still applies. The absolute extrema can only occur at endpoints or critical values. Make a list of those

important x-values		corresponding y-values
$x=-\sqrt{3}$	critical	$y=-4$
$x=0$	critical	$y=5$
$x=\sqrt{3}$	critical	$y=-4$

(no endpoints)

Conclusion: Absolute min is $y=-4$, occurs at $x=\sqrt{3}$ and $x=-\sqrt{3}$. But there is no absolute max.