Tuesday, October 29, 2013 (Day 37)

- Swipe your I.D.
- Pick up your graded work.
- Exam 3 tomorrow Covers Chapter 5.
  - Bring your O.U. I.D. to the exam

Today: Finishing Section 5-6 Optimization

Optimization Example #2 (similar to Suggested Exercise 5-6 #1)

Find positive numbers x, y such that

- their product is 9000
- The sum $10x + 25y$ is minimized

Solution

Step 1 Write an equation involving x & y that expresses the fact that their product is 9000.

solution $\boxed{x \cdot y = 9000 \quad \text{equation I}}$

Step 2 Write an equation involving x & y that expresses the information about the sum.

Solution Let S stand for the sum $10x+25y$.

That is $\boxed{S = 10x + 25y \quad \text{equation II}}$

Goal is to minimize the value of S.

Step 3 We want to eliminate the variable y.

Start by solving equation I for y in terms of x.

result: $y = \frac{9000}{x}$ new equation I

<u>Step #</u> Substitute Equation I into Equation II to eliminate y.

result $$S = 10x + 25\left(\frac{9000}{x}\right)$$

Notice: this equation
- only involves S and x
- it is solved for S in terms of x.

So this equation gives us S as <u>a function of</u> the variable x.

Use function notation to say that:

$$S(x) = 10x + \frac{25(9000)}{x}$$

Our goal is to find the value of x that minimizes the value of $S(x)$.

What is the Domain?

We're told that x, y must be positive numbers.

So the domain is all $x > 0$.

That is, the interval $(0, \infty)$

So our job is to ~~maximize~~ minimize

$$\$(x) = 10x + \frac{25(9000)}{x}$$

on the interval $(0, \infty)$

Step 5 Use calculus to solve the minimization problem.

Find critical values of $S(x)$

We will need $S'(x)$

It will help to first rewrite $S(x)$ in more useful form.

rewrite $S(x) = 10x + 25(9000)x^{-1}$

derivative $S'(x) = \frac{d}{dx}\left(10x + 25(9000)x^{-1}\right)$

$$= 10 + 25(9000)(-1)x^{(-2)}$$

$$S'(x) = 10 - \frac{25(9000)}{x^2}$$

Partition numbers for $S'(x)$

Are there any values of x that cause $S'(x)$ to not exist? yes $x=0$, because

$S'(0)$ DNE

Are there any values of x that cause $S'(x) = 0$? Answer by solving

$$10 - \frac{25(9000)}{x^2} = 0$$

$$10 = \frac{25(9000)}{x^2}$$

$$10x^2 = 25(9000)$$

$$x^2 = \frac{25(9000)}{10}$$

$$x^2 = 25(900)$$

So $x = \sqrt{(25)(900)} = \sqrt{25} \cdot \sqrt{900} = 5 \cdot 30 = 150$

So $x = 150$ is also a partition number for S'.

Partition numbers for $S'(x)$ are $x = 0$, $x = 150$.

Which of these are critical values for S?

Must see if $S(0)$ exists and if $S(150)$ exists.

Notice that $S(0)$ DNE so $x = 0$ is <u>not</u> critical value of S.

but $S(150) = 10(150) + \frac{25(9000)}{150}$. This <u>does</u> exist.

Conclude that $x = 150$ is the only critical value for $S(x)$.

Two ways to check that.

<u>Method #1</u> Make sign chart for $S'(x)$

<u>Method #2</u> Study $S''(x)$ to investigate <u>concavity</u>.

Let's do method #2

$$S'(x) = 10 - \frac{25(9000)}{x^2} = 10 - 25(9000)x^{-2}$$

$$S''(x) = \frac{d}{dx}\left(10 - 25(9000)x^{-2}\right)$$

$$= 0 - 25(9000)(-2)x^{-3}$$

$$= \frac{25(9000)}{2x^3}$$

Observe that for all $x > 0$, the values of $S''(x)$ will be positive.

So $S(x)$ is concave up for all $x > 0$.

Therefore, the critical value $x = 150$ is the location of an absolute min.

Step 6 Find corresponding value of y.

Solution Use equation 1

$$x \cdot y = 9000$$

$$y = \frac{9000}{x}$$

Substitute in $x = 150$

$$y = \frac{9000}{150} = 60$$

So use $x = 150, y = 60$

Step 7

Also find the value of the sum.

Solution

$$S(x) = 10x + \frac{25(9000)}{x} = \cdots = 9000$$

Another Example, this one involving an application.

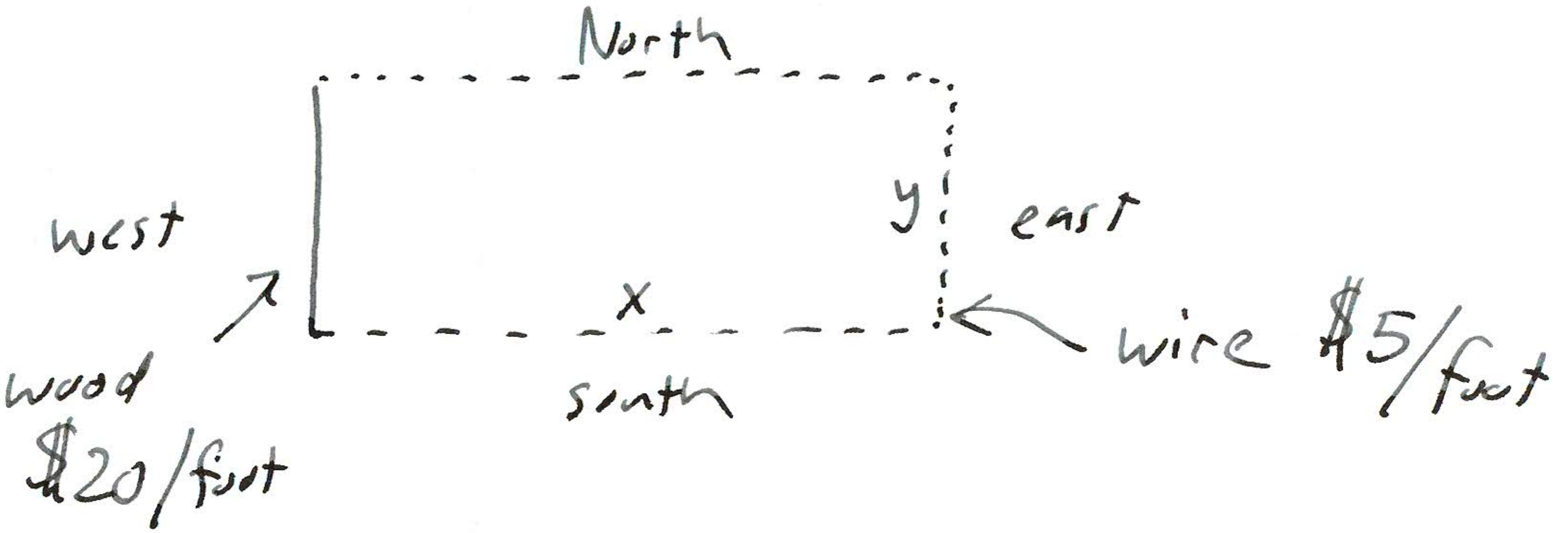
A farmer needs to build a rectangular fenced chicken yard. He wants an area of $9000\ \text{ft}^2$.

The west side of the fence faces the road, and must be made of wood. Wood fence costs \$20 per foot.

The north, east, south sides can be made from wire fence that costs \$5 per foot.

What are the dimensions of the cheapest fence? How much does it cost?

Solution Make a drawing



Need area of 9000 $\Longrightarrow$ $x \cdot y = 9000$

Cost of the fence C

$$C = \underbrace{x \cdot 5}_{\text{North}} + \underbrace{y \cdot 5}_{\text{east}} + \underbrace{x \cdot 5}_{\text{south}} + \underbrace{y \cdot 20}_{\text{west}}$$

$$C = 10x + 25y$$

Our job has become this

Find two numbers x, y that are positive and such that

$$x \cdot y = 9000$$

and such that the sum

$$C = 10x + 25y$$

is minimized.

We solved this problem earlier!

The result is $x = 150$, $y = 60$

The resulting cost is $C = 9000$