Tuesday, Nov 19, 2013 (Day 48)

Swipe your I.D.

Pick up your graded work

Exam Friday Covers Chapter 6

(Bring your I.D. (Sorry!))

Study the Substitution Method!!

Continuing the Fundamental Theorem of Calculus today, with applications of the fundamental theorem.

- Total Change Problems
- Average value of a function over an integral.

Total Change Problems

The form of the fundamental theorem that we have been using so far is

$$\int_{x=a}^{x=b} f(x)\,dx = F(b) - F(a)$$

In this expression, $f(x)$ is the integrand.

$F(x)$ is an antiderivative obtained by doing an indefinite integral.

$$\begin{tikzcd}[column sep=huge]
F(x) \arrow[r, bend left, "\frac{d}{dx}F(x)", "\text{Check}"'] & f(x) \arrow[l, bend left, "\text{Find } F"', "\int f(x)\,dx"]
\end{tikzcd}$$

Another form

use integrand $F'(x)$ in the Fundamental Theorem

$$\int_{x=a}^{x=b} F'(x)\,dx = F(b) - F(a)$$

$$\begin{tikzcd}[column sep=huge]
F(x) \arrow[r, bend left, "\frac{d}{dx}F(x)", "\text{check}"'] & F'(x) \arrow[l, bend left, "\text{find } F(x)", "\int F'(x)\,dx"']
\end{tikzcd}$$

integrand

"Total Change" refers to the ~~the~~ idea that the expression $F(b) - F(a)$ represents a change in F. (the "total change")

total change $= \Delta F = F(b) - F(a)$.

We will be doing problems where the stated goal is to find some total change ΔF.

That will lead us to doing the definite ~~fact~~ integral.

(Our previous uses of the Fundamental Theorem were in problems where we wanted to find a signed area.)

Example#1 of a Total Change Problem

A company makes & sells bikes.

Demand x is the number of bikes made in a month.
(units: bikes per month)

Cost $C(x)$ is the cost of producing that monthly batch of bikes (units: dollars per month)

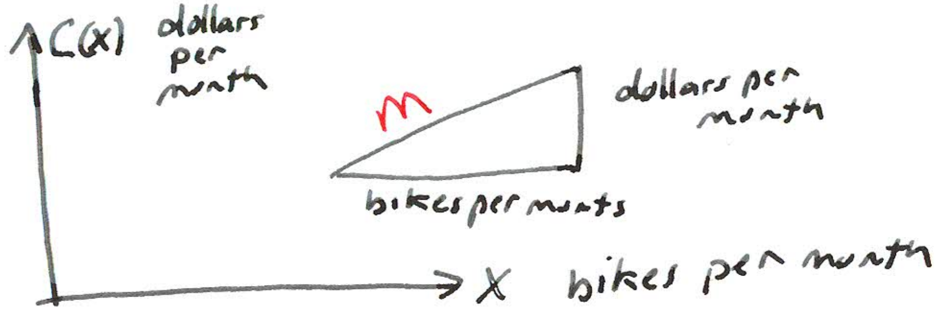
We don't know what $C(x)$ is.

Suppose we are given the marginal cost function

$$C'(x) = 1000 - \frac{x}{2} \quad \text{for} \quad 0 \le x \le 800$$

Remark

The units of C' ought to be



The slope m would have units

$$\frac{\text{dollars per month}}{\text{bikes per month}} = \frac{\text{dollars}}{\text{bike}} \quad \text{units of } C'$$

Note that the book says the units of C' are dollars.

We will therefore <u>ignore</u> the issue of units of C'.