Monday, November 25, 2013 (Day 50)

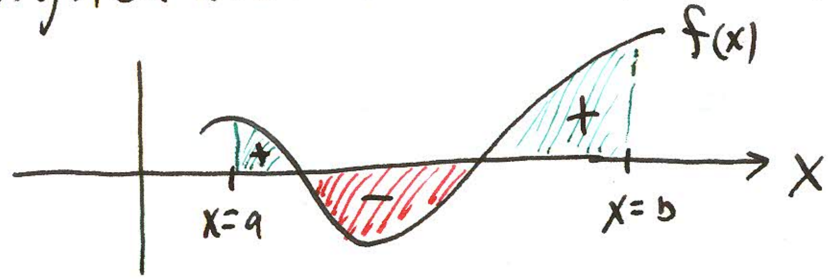
- Swipe your I.D.

Chapter 7 The Area Between Two Curves

Recall the Definition of

The Signed Area Under Graph of f from $x=a$ to $x=b$.

picture

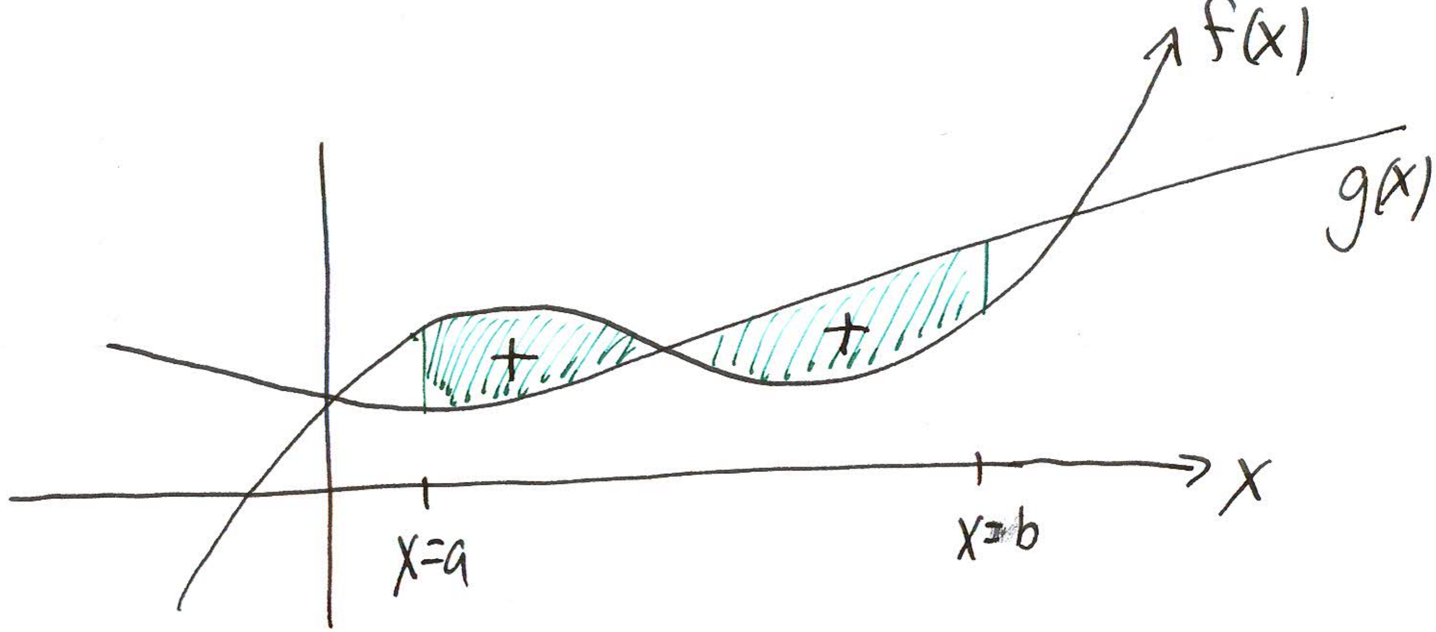


calculation: $SA = \int_{x=a}^{x=b} f(x)\,dx$ This definite integral takes care of the signs.

New Definition

"The area between graphs of $f(x)$ and $g(x)$ from $x = a$ to $x = b$."

Picture



All regions get plus signs. The "area between graphs of $f(x)$ and $g(x)$" means unsigned area

Question: How do we compute this area?

Answer in simple case where one of the graphs stays on top.

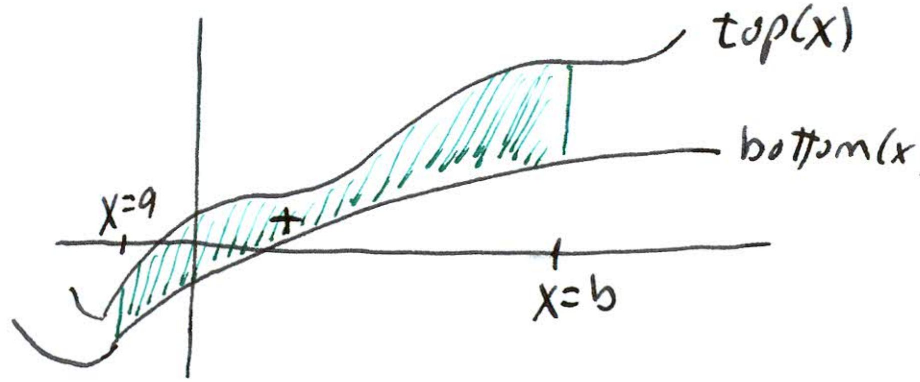
Theorem 1

If top(x) and bottom(x) are two functions that

- are continuous on the interval $a \leq x \leq b$
- have property that $\text{bottom}(x) \leq \text{top}(x)$ for $a \leq x \leq b$

Then the "area between graph of top(x) and bottom(x) from $x=a$ to $x=b$" is equal to the number obtained by doing this definite integral

$$USA = \int_{x=a}^{x=b} \text{top}(x) - \text{bottom}(x)\,dx$$



In practice, the definite integral is fairly <u>easy</u> to do. (we did Definite Integrals in Ch. 6) What is difficult is determining in a given problem whether or not we have a top(x) function and a bottom(x) function.

<u>Example #1</u>

Find the area between the graphs of $y = 2x - 2$ and $y = x^2 + 1$ from $x = -3$ to $x = 2$.

<u>Solution</u>: We would <u>like</u> to solve this by doing an integral

$$USA = \int_{x=-3}^{x=2} top(x) - bottom(x)\,dx.$$

In order to do that, we must determine if we have a top graph & bottom graph.

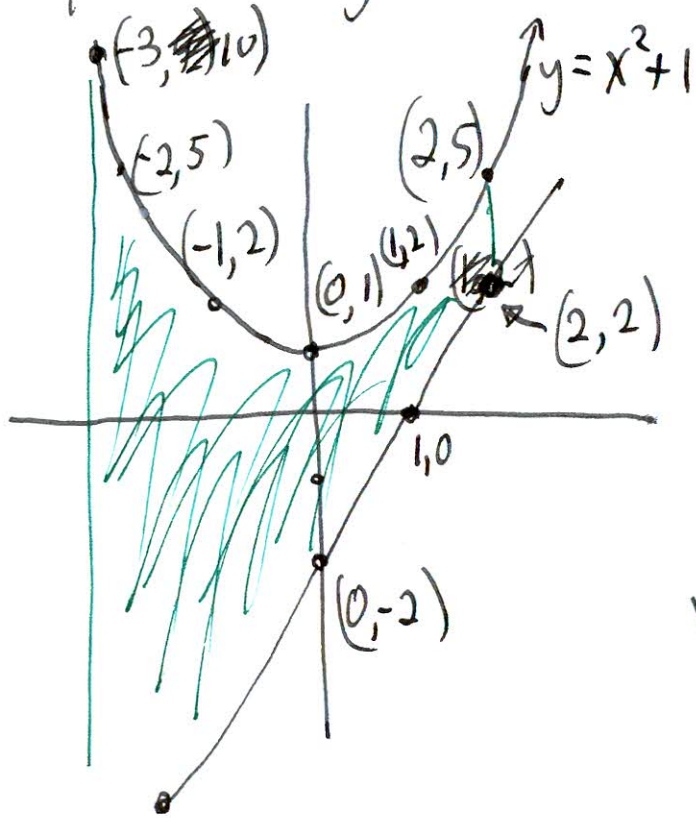
Graph of $y = 2x - 2$ is a straight line

Slope $m = 2$

y-intercept at $(x,y) = (0,-2)$

Graph of $y = x^2 + 1$ is a standard parabola moved up one unit

So vertex at $(x,y) = (0,1)$



Clearly, we can use

$\text{top}(x) = x^2 + 1$

$\text{bottom}(x) = 2x - 2$