## Reference 3: Facts About Limits from Section 3-1

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$ exist, where $L$ and $M$ are real numbers, then

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| **Theorem 2.1:** | $\lim_{x \to c} k = k$ |
| **Theorem 2.2:** | $\lim_{x \to c} x = c$ |
| **Theorem 2.3:** | $\lim_{x \to c}[f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x) = L + M$ |
| **Theorem 2.4:** | $\lim_{x \to c}[f(x) - g(x)] = \lim_{x \to c} f(x) - \lim_{x \to c} g(x) = L - M$ |
| **Theorem 2.5:** | If $k$ is a constant, then $\lim_{x \to c} kf(x) = k \lim_{x \to c} f(x) = kL$. |
| **Theorem 2.6:** | $\lim_{x \to c}\big(f(x) \cdot g(x)\big) = \left(\lim_{x \to c} f(x)\right) \cdot \left(\lim_{x \to c} g(x)\right) = L \cdot M$ |
| **Theorem 2.7:** | If $\lim_{x \to c} g(x) = M \neq 0$, then $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{L}{M}$ |
| **Theorem 2.8:** | If $n$ is a positive odd integer, or if $n$ is a positive even integer and $\lim_{x \to c} f(x) = L > 0$, then $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)}$. |
| **Theorem 3:** | If $f$ is a polynomial, then $\lim_{x \to c} f(x) = f(c)$. If $r$ is a rational function and $c$ is a real number that is in the domain of the function $r$, then $\lim_{x \to c} r(x) = r(c)$. |
| **Definition:** | If $\lim_{x \to c} f(x) = L = 0$ and $\lim_{x \to c} g(x) = M = 0$, then then $\lim_{x \to c} \frac{f(x)}{g(x)}$ is said to have the *indeterminate form* $\frac{0}{0}$. In this case, Theorem 2.7 cannot be used to determine the limit. |
| **Theorem 4:** | If $\lim_{x \to c} f(x) = L \neq 0$ and $\lim_{x \to c} g(x) = M = 0$, then $\lim_{x \to c} \frac{f(x)}{g(x)}$ does not exist according to the Definition of the Limit found in Section 3-1. (Remark: In Section 3-2, the Definition of Limit gets expanded, with the result that in some cases, we will say that the limit is infinity or negative infinity and will write $\lim_{x \to c} \frac{f(x)}{g(x)} = \infty$ or $\lim_{x \to c} \frac{f(x)}{g(x)} = -\infty$. But this will not always be the case. That is, sometimes the limit will not exist even with the expanded definition of limit.) |
| **One $x$ Rule (not in book)** | If $f$ and $g$ are functions whose $y$-values differ at only one $x$-value, $c$ (that is, if the $y$-values are the same for $x \neq c$ but differ at $x = c$), then $\lim_{x \to c} f(x) = \lim_{x \to c} g(x)$. |