Reference 4: Definitions of Rates of Change

Definition of *Average Rate of Change*

- **words:** the average rate of change of f as the input changes from a to b
- **usage:** f is a function that is continuous on the interval $[a, b]$.
- **meaning:** the number $m = \frac{f(b)-f(a)}{b-a}$
- **graphical interpretation:** The number m is the slope of the secant line that touches the graph of f at the points $\big(a, f(a)\big)$ and $\big(b, f(b)\big)$.
- **remark:** The average rate of change m is a number.

Definition of *Instantaneous Rate of Change*

- **words:** the instantaneous rate of change of f at a
- **alternate words:** the derivative of f at a
- **symbol:** $f'(a)$
- **meaning:** the number $m = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$
- **graphical interpretation:** The number m is the slope of the line tangent to the graph of f at the point $(x, y) = \big(a, f(a)\big)$.
- **remark:** The instantaneous rate of change $f'(a)$ is a number.

Definition of the *Derivative*

- **words:** the derivative of f
- **symbol:** f'
- **meaning:** f' is a function. To describe a function, one must show how it produces output for a given input. For an input x, the output is the number $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.
- **graphical interpretation:** For an input x, the output $f'(x)$ is the number that is the slope of the line tangent to the graph of f at the point $(x, y) = \big(x, f(x)\big)$.
- **remark:** The derivative f' is a function.

Terminology of *Position* and *Velocity*

- **Time:** When our book uses mathematical functions to describe the motion of objects, x is a variable that represents the elapsed time.
- **Position:** To say an object is *"moving in 1 dimension"* means that it can go forward or backward in one direction but cannot turn. In such situations, a single coordinate can be used to keep track of the position of the object. A function called the *position function* gives the value of the coordinate at a given time. In our book, the position function is called f. That is, at time x, the coordinate of the object is the number $f(x)$.
- **average velocity:** The words *"the average velocity from time $x = a$ to time $x = b$"* mean the same thing as *"the average rate of change of position from time $x = a$ to time $x = b$"*. That is, the number $m = \frac{f(b)-f(a)}{b-a}$.
- **instantaneous velocity:** The words *"instantaneous velocity at time $x = a$"* mean the same thing as *"instantaneous rate of change of position at time $x = a$."* That is, the number $m = f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$.
- **velocity:** The word *"velocity"* means the same thing as the words *"derivative of the position function"*. That is, the velocity is the function f'.

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