														,											
L	A	S	T		N	A	M	E						,	F	I	R	S	T		N	A	M	E	

Cover Sheet for 2013-2014 Spring Semester MATH 3210/5210 (Barsamian) Homework 8

(Due Wednesday, April 9, 2014. Staple this page to the front of your work.)

Problem:	1	2	3	4	5	Total	Rescaled
Your Score:							
Possible:	20	20	20	20	20	100	10

For all of the problems on this assignment, you must show all details of the calculations. It would be a good idea to check your answers by using an online tool such as Wolfram Alpha. But remember that you must show all of the details that lead to the solution, and you should present the steps clearly. The idea is not to turn in scrap paper work, but rather to turn in clear steps that somebody else can read and understand.

(Three problems similar to suggested exercise 5-1#3(a),(b),(c))
For each matrix $A \in M_{n\times n}(F)$

(i) Determine all the eigenvalues of A. Show all details of the calculation.
(ii) For each eigenvalue λ of A, find the eigenvectors corresponding to λ.
(iii) If possible, find a basis for F^n consisting of eigenvectors of A.
(iv) If successful in finding such a basis, determine an invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ = D$.

You must show all details of the calculations. (It would be a good idea to check your work by using an online tool such as Wolfram Alpha. But remember that you must show all of the details that lead to the solution, and you should present the steps clearly.

[1] (20 points) Let $A = \begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix} \in M_{3\times 3}(R)$

[2] (20 points) Let $A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \in M_{2\times 2}(C)$

[3] (20 points) Let $A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \in M_{2\times 2}(R)$

[4] (20 points) Let $V = R^2$ and define $T: V \to V$ by $T(a,b) = (a+b, 4a+b)$.
Find the eigenvalues of T and find an ordered basis β for V such that $[T]_\beta$ is diagonal.

[5] (20 points) Let $V = P_2(R)$ and define $T: V \to V$ by $T\big(f(x)\big) = xf'(x) + f(2)x + f(3)$.
(a) Let $\alpha = \{v_1, v_2, v_3\} = \{1, x, x^2\}$, the standard basis for $P_2(R)$. Find $[T]_\alpha$.
(b) Find the eigenvalues of the matrix $[T]_\alpha$ and the corresponding eigenvectors of $[T]_\alpha$. These eigenvectors should be column vectors representing elements of R^3.
(c) Using your result from (b), find a corresponding ordered basis β for$P_2(R)$ such that $[T]_\beta$ is a diagonal matrix. (The basis vectors in β should be polynomials, elements of $P_2(R)$.)