L	A	S	T		N	A	M	E						,	F	I	R	S	T		N	A	M	E	

Cover Sheet for 2014-2015 Fall Semester MATH 3200/5200 (Barsamian) Homework 4
(Due at the start of class Monday, September 29, 2014. Staple this cover sheet to the front of your work.)

Problem:	1	2	3	4	5	Total	Rescaled
Your Score:							
Possible:	20	20	20	20	20	100	10

Reading: In Chapter Two, read

Section II, Linear Independence, pages 102 – 110
Section III.1, Basis, pages 114 – 118
Section III.2, Dimension, pages 120 – 125
Section III.3 Vector Spaces and Linear Systems, pages 127 - 132

Suggested Exercises: (These 38 exercises are not to be turned in and are not graded, but you should do as many of them as possible and keep your solutions in a notebook for study. Note that detailed solutions to all of the Suggested Exercises are available in the solutions manual provided for free on the author's web site.)

Two.II.1 Exercises # 10, 21, 23, 26, 27, 29, 29, 34, 41ac (from pages 110 - 114)
Two.III.1 Exercises # 18, 19, 20, 21, 22, 23, 26, 27, 29, 33, 35, 36 (from pages 118 - 120)
Two.III.2 Exercises # 18, 19, 23, 24, 25, 29, 31 (from pages 125 - 126)
Two.III.3 Exercises # 19, 20, 21, 23, 24, 25, 27, 35, 36, 38 (from pages 132 - 134)

Assigned Exercises: Turn in your solutions to the following five exercises, with this cover sheet stapled to the front of your work.

[1] Decide whether each subset A, B, C, D of $\mathbb{R}^3$ is linearly dependent or linearly independent. Show all details.

$$A = \left\{\begin{pmatrix}4\\5\\3\end{pmatrix}, \begin{pmatrix}2\\2\\1\end{pmatrix}, \begin{pmatrix}-1\\1\\1\end{pmatrix}\right\},\ B = \left\{\begin{pmatrix}3\\1\\2\end{pmatrix}, \begin{pmatrix}2\\1\\4\end{pmatrix}, \begin{pmatrix}5\\2\\6\end{pmatrix}\right\},\ C = \left\{\begin{pmatrix}2\\1\\3\end{pmatrix}, \begin{pmatrix}4\\2\\5\end{pmatrix}\right\},\ D = \left\{\begin{pmatrix}4\\5\\3\end{pmatrix}, \begin{pmatrix}2\\2\\1\end{pmatrix}, \begin{pmatrix}-1\\1\\1\end{pmatrix}, \begin{pmatrix}5\\2\\6\end{pmatrix}\right\}.$$

Decide whether each subset E, F, G, H of $\mathcal{P}_2$ is linearly dependent or linearly independent. Show all details.

$$E = \{2 - x - x^2, 2 + x, 7 + 5x + x^2\}$$
$$F = \{2 - x - x^2, 7 + 5x + x^2\}$$
$$G = \{1 + 2x^2, 6 - 3x + 9x^2, 3 - x + 5x^2\},$$
$$H = \{2 - x - x^2, 2 + x, 7 + 5x + x^2, 1 + 2x + 3x^2\}$$

[2] (a) Show that any set of four vectors in $\mathbb{R}^3$ is linearly dependent.
(b) What is the greatest number of elements that a linearly independent subset of $\mathbb{R}^3$ can have? Explain.

[3] Decide if each is a basis for $\mathbb{R}^3$. Show all details clearly and explain.

$$(a)\ \left\langle \begin{pmatrix}4\\5\\3\end{pmatrix}, \begin{pmatrix}2\\2\\1\end{pmatrix}, \begin{pmatrix}-1\\1\\1\end{pmatrix} \right\rangle \quad (b)\ \left\langle \begin{pmatrix}3\\1\\2\end{pmatrix}, \begin{pmatrix}2\\1\\4\end{pmatrix}, \begin{pmatrix}5\\2\\6\end{pmatrix} \right\rangle \quad (c)\ \left\langle \begin{pmatrix}2\\1\\3\end{pmatrix}, \begin{pmatrix}4\\2\\5\end{pmatrix} \right\rangle \quad (d)\ \left\langle \begin{pmatrix}4\\5\\3\end{pmatrix}, \begin{pmatrix}2\\2\\1\end{pmatrix}, \begin{pmatrix}-1\\1\\1\end{pmatrix}, \begin{pmatrix}5\\2\\6\end{pmatrix} \right\rangle$$

[4] Using the basis $D = \langle 1, 1 + x, 1 + x + x^2, 1 + x + x^2 + x^3 \rangle$ for $\mathcal{P}_3$, represent vector $a + bx + cx^2 + dx^3 \in \mathcal{P}_3$ with respect to the basis D. Show all details clearly.

[5] Find the span of each subset of $\mathcal{P}_2$ and then find a basis for that span. Show all details clearly.

$$A = \{1 + 2x, 3 + 4x\} \subset \mathcal{P}_2$$
$$B = \{1 + 2x, 3 + 4x^2\} \subset \mathcal{P}_2$$